ON PETRI NETS WITH DETERMINISTIC AND EXPONENTIALLY DISTRIBUTED FIRING TIMES

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ABSTRACT
A class of Petri nets (DSPN) in which transitions can fire after either a deterministic or a random, exponentially distributed, firing delay is defined, and a solution technique is presented to obtain the steady-state probability distribution over markings, introducing restrictions on the use of deterministic firing delays. An example of application of this modeling technique is presented to demonstrate the impact that the use of a mix of deterministic and exponentially distributed firing delays (instead of all exponentially distributed firing delays) can have on performance and reliability estimates.

1. INTRODUCTION

The most natural way of introducing time into a Petri net (PN) is based on the interpretation of a PN as a system model in which, given a system situation (marking), some time must elapse before an event occurs (a transition fires). The event is the final result of some activity that is performed by the system when it is in the situation specified by the marking. Time is thus naturally associated with transitions, indicating that they can fire some time after they become enabled. The choice of associating time with transitions is the most frequent in the literature on timed PN (TPN) [1] and the resulting models are known as timed transition PN or TTPN. We always assume that in a TTPN the firing of a transition is an atomic operation, and that transitions fire one at a time.

From TTPN it is possible to derive either analytical or simulation models of the system under investigation. The derivation of a simulation model from a TTPN poses mainly implementation problems, and some packages exist for this purpose. The derivation of an analytical model from a TTPN may instead be more complex and present subtle theoretical problems. In this paper we only consider the issues related to the TTPN analysis and do not discuss the problems concerning their simulation.

Traditionally, timing in a TTPN was specified either in a deterministic or in a stochastic manner. In the former case times are defined to be constant, and the TTPN analysis can be performed using an algebraic approach. Timing constrains in this case usually change the qualitative behavior of the model with respect to the underlying untimed PN. In the latter case, transition firing delays are defined to be random variables with given probability distributions, so that the TTPN can be viewed as a graphical representation of a stochastic process, that can be analyzed using a probabilistic approach. The qualitative behavior of the model does not change with
respect to the underlying PN, provided that time distributions have unlimited support.

When random firing delays are used, it is necessary to further separate the cases of discrete- and continuous-time distributions. Some results concerning the integration of deterministic transition firing delays into TTPN with geometrically distributed timing have already been published [2, 3]. In the first case [2] the deterministic delays are restricted to be equal to the step of the geometric distribution, whereas in the second case [3] arbitrary deterministic times are allowed.

In this paper we discuss for the first time the problem of the integration of deterministic times into TTPN in which timed transitions are associated with random firing delays having continuous distributions. This work can be considered as a step toward the extension of the class of allowed distributions for continuous-time TTPN. Indeed, originally in TTPN that used continuous-time random variables as transitions firing delays, the only allowed distribution was the exponential that, due to its memoryless property, leads to an isomorphism between TTPN and continuous-time Markov chains. This highly simplifies the model analysis since a wide gamut of results are readily available in this case. However, exponential timing may in some cases imply a gross approximation of the system characteristics. The class of TTPN that are presented in this paper permits the association of deterministic and exponentially distributed firing delays with transitions. Under the restriction that no more than one deterministic transition is enabled in any marking, these models can be analyzed using results of the theory of semi-Markov processes.

The paper is organized as follows. In Section 2, after a brief review of the types of timing in continuous-time that were proposed in the literature, we discuss the use of both deterministic and exponential distributions in the same TTPN. In Section 3 the steady-state solution technique for this new class of TTPN (which will be referred to as Deterministic and Stochastic TTPN, or DSPN for short) is formalized, and in Section 4 a very simple example is solved in closed form to clarify the solution technique. Finally, in Section 5 we present an example of application of DSPN to the numerical analysis of a fault-tolerant multiprocessor system in which processors are subject to failures and repairs, to show the impact of the introduction of a mix of deterministic and stochastic timing with respect to an all-exponential approximation.

2. STOCHASTIC PETRI NETS

TTPN in which firing delays are defined to be random variables with continuous probability distributions are generally called Stochastic Petri Nets.

2.1. TTPN With Exponential Timing (SPN)

Several authors independently proposed the association of random, exponentially distributed firing delays with PN transitions as a simple but useful technique to augment a PN model with timing. In the earlier works [4, 5] the authors mainly considered the association of a (possibly marking dependent) firing rate with each transition in the net. The resulting models were called Stochastic PN (SPN). An isomorphism exists between SPN models and continuous-time Markov chains (MC). Each marking of a SPN maps into a state of the MC. The infinitesimal generator of the MC isomorphic to the SPN can be easily obtained, and the steady-state probability distribution over the MC states can then be computed. The limitation of SPN is that the graphical representation of systems becomes rapidly difficult when system size and complexity increase. Moreover, the number of states of the associated MC grows very fast with the dimensions of the net. SPN can thus be used only to model limited size systems.