Circuit Definitions of Nondeterministic Complexity Classes

H. Venkateswaran
Department of Computer Science and Automation
Indian Institute of Science, Bangalore-560 012, INDIA

Abstract

We consider restrictions on Boolean circuits and use them to obtain new uniform circuit characterizations of nondeterministic space and time classes. We also obtain characterizations of counting classes based on nondeterministic time bounded computations on the arithmetic circuit model. It is shown how the notion of semi-unboundedness unifies the definitions of many natural complexity classes.

1 Introduction

Uniform Boolean circuits have provided a very useful framework to study some of the important issues that arise in Turing machine based complexity theory. Close connections have been established between complexity classes based on uniform circuits with those based on the machine model [1,5,9,10,12]. In one direction, complexity classes defined using the circuit model have been characterized using the machine model. NC is a well known example of such a complexity class defined using the uniform Boolean circuit model [9] that has been characterized using the alternating Turing machine model by Ruzzo [12]. In the other direction, traditional complexity classes based on the machine model have been characterized in the circuit model. The definition of the class P using Boolean circuits [7,10] is probably the first such result. Other results of this nature are the characterizations in the circuit model of the classes AC^1 [13] and LOGCFL [14]. The results by Ruzzo [12] also make it possible to obtain circuit characterizations of complexity classes defined using alternating Turing machines. The work reported here extends these results to characterize classes defined using nondeterministic Turing machines.

First, we consider restrictions of Boolean circuits and use them to characterize nondeterministic space and time classes. This includes a characterization of nondeterministic time classes on the semi-unbounded fan-in circuit model. Semi-unbounded
fan-in circuits, which are Boolean circuits in which the AND gates have bounded fan-in, have been previously used to define the class LOGCFL [14]. We define skew circuits as Boolean circuits in which all but one input of every AND gate are circuit inputs and use them to characterize nondeterministic space and time classes. Nondeterministic space is defined in terms of the size of such circuits and nondeterministic time is shown to correspond to the depth of these circuits. This should be contrasted with the well known correspondences between deterministic time and Boolean circuit size [10] and between nondeterministic space and Boolean circuit depth [1].

Second, we use the monotone arithmetic circuit model to characterize counting classes based on nondeterministic time bounded computations. Monotone arithmetic circuits are arithmetic circuits over the domain of non-negative integers and which use only the addition and multiplication operations. An interesting consequence of this characterization is the definition of the well known counting class \#P as the set of functions computed by uniform families of monotone arithmetic circuits that have polynomial depth and polynomial degree. The degree measure here refers to the algebraic degree of the polynomial associated with the circuit.

Some of the appealing features of the characterization results in this paper are listed below.

* The circuit characterizations of NP presented here are, to our knowledge, the first uniform circuit characterizations of this important complexity class. Of particular interest is the definition of NP as the class of languages accepted by uniform families of semi-unbounded fan-in circuits of exponential size and log depth. This provides a framework to study some interesting questions about the class NP. Recently, Borodin et al. [2] proved that if a language is accepted by a family of semi-unbounded fan-in circuits of size \(Z(n)\) and depth \(D(n)\), then its complement is accepted by a family of semi-unbounded fan-in circuits of size polynomial in \(Z(n)\) and depth \(O(D(n)\log Z(n))\). Their result does not apply directly to NP, since it only shows that CO-NP is accepted by semi-unbounded fan-in circuits of exponential size and polynomial depth. But, it does raise a relevant question: are classes accepted by size \(Z\) and depth \(O(\log Z)\) semi-unbounded fan-in circuits closed under complement? It is known that the classes accepted by size \(Z\) and depth \(O(\log n)\) semi-unbounded fan-in circuits are not closed under complement [14]. Another complexity question pertaining to NP that can be phrased in this model is its relationship with the other classes definable using semi-unbounded fan-in circuits. Thus, for instance, the separation between NP and LOGCFL now becomes a question of the relative power of exponential size and polynomial size semi-unbounded fan-in circuits of logarithmic depth.