Generalizing Lambert's Law For Smooth Surfaces

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Abstract. One of the most common assumptions for recovering object features in computer vision and rendering objects in computer graphics is that diffuse reflection from materials is Lambertian. This paper shows that there is significant deviation from Lambertian behavior in diffuse reflection from smooth surfaces not predicted by existing reflectance models, having an important bearing on any computer vision technique that may utilize reflectance models including shape-from-shading and binocular stereo. Contrary to prediction by Lambert's Law, diffuse reflection from smooth surfaces is significantly viewpoint dependent, and there are prominent diffuse reflection maxima effects occurring on objects when incident point source illumination is greater than 50° relative to viewing including the range from 90° to 180° where the light source is behind the object with respect to viewing. Presented here is a diffuse reflectance model, derived from first physical principles, utilizing results of radiative transfer theory for subsurface multiple scattering together with Fresnel attenuation and Snell refraction at a smooth air-dielectric surface boundary. A number of experimental results are presented demonstrating striking deviation from Lambertian behavior predicted by the proposed diffuse reflectance model.

1 Introduction

A prevalent class of materials encountered both in common experience and in computer vision/robotics environments are inhomogeneous dielectrics which include plastics, ceramics, and, rubber. In computer vision a widely used assumption about diffuse reflection from materials is Lambert's law [13], namely the expression:

\[ \frac{1}{\pi} L \rho \cos \psi d\omega \]

where light is incident with radiance \( L \), at incidence angle \( \psi \), and reflected through a small solid angle \( d\omega \), and \( \rho \) is termed the diffuse albedo in the range \([0, 1.0]\). This reflectance model is typically instantiated into the implementation of a large number of algorithms such as shape-from-shading [9] and photometric-based binocular stereo [6], [19]. It is therefore important for researchers in the computer vision community who utilize assumptions about diffuse reflection to be aware of the conditions under which there is significant deviation from Lambert's law.
Almost all diffuse reflection from inhomogeneous dielectrics physically arises from subsurface multiple scattering of light caused by subsurface inhomogeneities in index of refraction. In this paper we model inhomogeneous dielectric material as a collection of scatterers contained in a uniform dielectric medium with index of refraction different from that of air. An expression is derived for diffuse reflected radiance resulting from the process of incident light refracting into the dielectric medium across a smooth surface boundary, producing a subsurface diffuse intensity distribution from multiple internal scattering, and then refraction of this subsurface diffuse intensity distribution back out into air. See Figure 1. Also accounted for is the infinite progression of internal specular reflection at the air-dielectric boundary and sub-surface scattering. A common property of diffuse reflection from smooth inhomogeneous dielectric surfaces is that such reflection is azimuth independent with respect to viewing about the surface normal, regardless of the fixed direction of incident light. We formally derive and empirically verify that for smooth inhomogeneous dielectric surfaces that exhibit such azimuth symmetric diffuse reflection, that

$$qL \times (1 - F(\psi, n)) \times \cos \psi \times (1 - F(\sin^{-1}\left(\frac{\sin\phi}{n}\right), 1/n)) \, d\omega$$

(1)

describes the reflected radiance into viewing angle, $\phi$, (i.e., angle between viewing and the surface normal, also known as emittance angle). The terms $F(\cdot)$ refer to the Fresnel reflection coefficients [18], $n$, is the index of refraction of the dielectric medium, and, $q$, is the total diffuse albedo. We show that the total diffuse albedo, $q$, is directly related to both the single scattering albedo describing the proportion of energy reradiated upon each subsurface single scattering, and, the index of refraction $n$. An initial first order derivation of expression 1 was presented in [23], however without the accounting for higher order effects which is presented below.

Particularly useful to object feature extraction in computer vision, our expression 1 for diffuse reflection allows precise characterization of the conditions under which the Lambertian model breaks down for inhomogeneous dielectrics and where our more accurate model should be used. We show that Lambert's law is valid for smooth dielectrics to within 5% only as long as both angle of incidence, $\psi$, and viewing angle, $\phi$, are simultaneously less than 50°. This means that for applications in computer vision there are a large number of situations in which Lambert’s law is significantly in error for smooth surfaces—near the occluding contour of objects under any illumination condition; for illuminations