The Reception of Gödel's Incompleteness Theorems

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"Die Arbeit über formal unentscheidbare Sätze wurde wie ein Erdbeben empfunden; insbesondere auch von Carnap."—(Popper, 1980).

"Kurt Gödel's achievement in modern logic ... is a landmark which will remain visible far in space and time."—John von Neumann[1]

It is natural to invoke geological metaphors to describe the impact and the lasting significance of Gödel's incompleteness theorems. Indeed, how better to convey the impact of those results—whose effect on Hilbert's program was so devastating and whose philosophical reverberations have yet to subside—than to speak of tremors and shock waves? The image of shaken foundations is irresistible.

Yet to adopt such seismic imagery is to suggest that the aftermath of intellectual upheaval is comparable to that of geological cataclysm: a period of slow rebuilding, preceded initially by widespread confusion and despair, or perhaps determined resistance; and though we might expect Gödel's discoveries to have provoked just such reactions, according to most commentators they did not. Thus van Heijenoort states "although [Gödel's paper] caused some momentary surprise, its results were soon widely accepted" (van Heijenoort, 1967, p. 594). Similarly, Kreisel has averred that "expected objections never materialized" (Kreisel, 1979, p. 13), and Kleene, speaking of the second incompleteness theorem (whose proof was only sketched in Gödel's (1931)), has even claimed "it seems no one doubted it" (Kleene, 1976, p. 767).

If these accounts are correct, one of the most profound discoveries in the history of logic and mathematics was assimilated promptly and almost without objection by Gödel's contemporaries—a circumstance so remarkable that it demands to be accounted for. The received explanation seems to be that Gödel, sensitive to the philosophical climate of opinion and anticipating objections to his work, presented his results with such clarity and rigor as to

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render them incontestable, even at a time of fervid debate among competing mathematical philosophies. The sheer force of Gödel's logic, as it were, swept away opposition so effectively that Gödel abandoned his stated intention of publishing a detailed proof of the second theorem (Gödel, 1931, p. 198).

On the last point there can be no dispute, as Gödel stated explicitly to van Heijenoort that "the prompt acceptance of his results was one of the reasons that made him change his plan" (van Heijenoort, 1967, ftn. 68a, p. 616). We may question, however, to what extent Gödel's subjective impression reflected objective circumstances. We must also recognize the hazard in assessing the cogency of Gödel's arguments from our own perspective. To be sure, the exposition in Gödel (1931) *now* seems clear and compelling; the proofs strike us as detailed but not intricate. But did it seem so at the time? After all, arithmetization of syntax was then a novel device, and logicians were not then so accustomed to the necessity for making precise distinctions between object- and metalanguage. Indeed, J. Barkley Rosser (who himself contributed to the improvement of Gödel's results) has observed that only *after* Gödel's paper appeared did logicians realize how careful they had to be (cf. Grattan-Guinness, 1981, ftn. 3, p. 499); precisely *because* of Gödel's results we no longer share the formalists' naive optimism, and so we are likely to be more receptive to Gödel's ideas.

Our faith in the efficacy of logic as a tool for overcoming intellectual resistance should also be tempered by consideration of the reception accorded other "paradoxical" results. For example, the Löwenheim–Skolem theorem, first enunciated by Löwenheim in 1915 and established with greater precision and broader generality by Skolem in a series of papers from 1920 to 1929, is certainly less profound than Gödel's discovery (with which, however, it is still sometimes confused)—yet it generated widespread misunderstanding and bewilderment, even as late as December 1938. (See, in particular, the discussion in Gonseth (1941, pp. 47–52).)

In what follows, I shall examine the reaction to Gödel's theorems in some detail, with the aim of showing that there *were* doubters and critics, as well as defenders and rival claimants to priority. (Of course, there were also some who *accepted* Gödel's results without fully *understanding* them.)

1. 1930: Announcement at Königsberg

Elsewhere (Dawson, 1984) I have described in detail the circumstances surrounding Gödel's first public announcement of his incompleteness discovery. In summary, the event occurred during a discussion on the foundations of mathematics that took place in Königsberg, 7 September 1930, as one of the final sessions of the Second Conference on Epistemology of the Exact Sciences organized by the *Gesellschaft für empirische Philosophie*. At the time, Gödel was virtually unknown outside Vienna; he had come to the conference to deliver a 20-minute talk on the results of his dissertation, completed the