3 Temporal Extension of Relational Algebra

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Abstract Temporal relational operations can be considered as temporal extensions of regular relational operations, which are the basic contents of temporal database technology. Like traditional relational operations, temporal relational operations can also be classified into two types: temporal relational algebra and temporal relational calculus. The former takes relation as a set of tuples, while the latter uses temporal predicates to select temporal tuples, which should meet not only the requirements in terms of attributes but also temporal predicates. In this chapter, first the regular relational operations are reviewed. Subsequently, temporal relational operations based on historical relational database model (HRDM) and bitemporal conceptual data model (BCDM) are discussed. Finally, three important properties—snapshot reducibility, temporal semi-completeness, and temporal completeness are introduced.

Keywords temporal relational algebra, temporal relational calculus, HRDM, BCDM, snapshot reducibility, temporal semi-completeness, temporal completeness

Temporal relational operations are the basic contents of temporal database technology, which can be considered as temporal extension of regular relational operations. Like regular relational operations, temporal relational operations can be classified into two types according to their theoretic basis: \textit{temporal relational algebra} and \textit{temporal relational calculus}. These two kinds of temporal operations are equivalent in expressive power (Zhang 1993).

Relational operations are important theoretic basis for query languages. Whenever relational database has been extended to temporal relational database, temporal relational operations have drawn many researchers’ attention.

James Clifford and Albert Croker (1987) proposed the historical relational data model (HRDM) and defined a temporal relational algebra based on it. Frank and

HRDM and BCDM are two representative models among temporal database models. We will discuss historical relational algebra and bitemporal relational algebra in this chapter.

Before we discuss temporal algebra, we should review regular algebra and calculus.

### 3.1 Regular Relational Operations

Regular relational operations (Codd 1970) are the basis of temporal relational operations. The primitive operations in regular relational database include relational algebra and relational calculus. Relational algebra is essentially equivalent in expressive power to relational calculus (and thus first-order logic). This result is known as Codd’s theorem (Codd 1972). Before we review regular relational algebra and regular relational calculus, some basic notions should be given here.

#### 3.1.1 Basic Notions

**Definition 3.1** Domain is the set of values allowed in an attribute.

For example, \{1, 2, 3, 4, 5\} and \{male, female\} are two different domains.

**Definition 3.2** Let \(D_1, D_2, \ldots, D_n\) be \(n\) sets. They may or may not be the same. Cartesian product of \(D_1, D_2, \ldots, D_n\) is defined as follows:

\[
D_1 \times D_2 \times \cdots \times D_n = \{(d_1, d_2, \ldots, d_n) | d_i \in D_i, i = 1, 2, \ldots, n\}
\]

(3.1)

\((d_1, d_2, \ldots, d_n)\) is an \(n\)-tuple, where \(d_i\) is an element of it.

For example, \(D_1 = \{1, 3\}\), \(D_2 = \{2, 4\}\), thus the Cartesian product of \(D_1\) and \(D_2\) is defined as: \(D_1 \times D_2 = \{(1,2),(3,2),(1,4),(3,4)\}\).

**Definition 3.3** Any subset of \(D_1 \times D_2 \times \cdots \times D_n\) is called a relation, which is a set of \(n\)-tuples, where \(n\) is the degree of the relation.