Chapter 13
A Fast Functional Locally Modeled Conditional Density and Mode for Functional Time-Series

Jacques Demongeot, Ali Laksaci, Fethi Madani, Mustapha Rachdi

Abstract We study the asymptotic behavior of the nonparametric local linear estimation of the conditional density of a scalar response variable given a random variable taking values in a semi-metric space. Under some general conditions on the mixing property of the data, we establish the pointwise almost-complete convergence, with rates, of this estimator. Moreover, we give some particular cases of our results which can also be considered as novel in the finite dimensional setting: Nadaraya-Watson estimator, multivariate data and the independent and identically distributed data case. On the other hand, this approach is also applied in time-series analysis to the prediction problem via the conditional mode estimation.

13.1 Introduction

Let (X_i, Y_i) for $i = 1, \ldots, n$ be n pairs of random variables that we assume are drawn from the pair (X, Y) which is valued in $\mathscr{F} \times \mathbb{R}$, where $\mathscr{F}$ is a semi-metric space equipped with a semi-metric d.

Furthermore, we assume that there exists a regular version of the conditional probability of Y given X, which is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$ and admits a bounded density, denoted by f^x. Local polynomial

Jacques Demongeot
Université J. Fourier, Grenoble, France, e-mail: Jacques.Demongeot@imag.fr

Ali Laksaci
Université Djillali Liabès, Sidi Bel Abbès, Algeria e-mail: alilak@yahoo.fr

Fethi Madani
Université P. Mendès France, Grenoble, France, e-mail: Fethi.Madani@imag.fr

Mustapha Rachdi
Université P. Mendès France, Grenoble, France, e-mail: Mustapha.Rachdi@upmf-grenoble.fr

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smoothing is based on the assumption that the unknown functional parameter is smooth enough to be locally well approximated by a polynomial (cf. Fan and Gijbels, 1996).

In this paper, we consider the problem of the conditional density estimation by using a locally modeling approach when the explanatory variable X is of functional kind and when the observations $(X_i,Y_i)_{i\in\mathbb{N}}$ are strongly α-mixing (cf. for instance, Rio (2000), Ferraty and Vieu (2006), Ferraty et al. (2006) and the references therein). In functional statistics, there are several ways of extending the local linear ideas (cf. Barrientos-Marin et al. (2009), Baìllo and Grané (2009), Demongeot et al. (2010), El Methni and Rachdi (2011) and the references therein). Here we adopt the fast functional locally modeling, that is, we estimate the conditional density f^x by $\widehat{a}$ which is obtained by minimizing the following quantity:

$$\min_{(a,b)\in\mathbb{R}^2}\sum_{i=1}^{n}\left(h_H^{-1}H(h_H^{-1}(y-Y_i))-a-b\beta(X_i,x)\right)^2 K(h_K^{-1}\delta(x,X_i)) \tag{13.1}$$

where $\beta(.,.)$ is a known bi-functional operator from $\mathscr{F}^2$ into $\mathbb{R}$ such that, $\forall\xi\in\mathscr{F}$, $\beta(\xi,\xi)=0$, with K and H are kernels and $h_K=h_{K,n}$ (respectively $h_H=h_{H,n}$) is chosen as a sequence of positive real numbers and $\delta(.,.)$ is a function from $\mathscr{F}^2$ into $\mathbb{R}$ such that $|\delta(.,.)|=d(.,.)$. Clearly, by simple algebra, we get explicitly the following definition of $\widehat{f^x}$:

$$\widehat{f^x}(y)=\frac{\sum_{i,j=1}^{n}W_{ij}(x)H(h_H^{-1}(y-Y_i))}{h_H\sum_{i,j=1}^{n}W_{ij}(x)} \tag{13.2}$$

where

$$W_{ij}(x)=\beta(X_i,x)\left(\beta(X_i,x)-\beta(X_j,x)\right)K(h_K^{-1}\delta(x,X_i))K(h_K^{-1}\delta(x,X_j))$$

with the convention $0/0=0$.

13.2 Main results

In what follows x denotes a fixed point in $\mathscr{F}$, N_x denotes a fixed neighborhood of x, S will be a fixed compact subset of $\mathbb{R}$, and $\phi_x(r_1,r_2)=\mathbb{P}(r_2\le\delta(X,x)\le r_1)$.

Our nonparametric model will be quite general in the sense that we will just need the following assumptions:

(H1) For any $r>0$, $\phi_x(r):=\phi_x(-r,r)>0$.

(H2) The conditional density f^x is such that $\exists b_1>0$, $b_2>0$, $\forall(y_1,y_2)\in S^2$ and $\forall(x_1,x_2)\in N_x\times N_x$: