Chapter 5
Robust Nonparametric Estimation for Functional Spatial Regression

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Abstract This contribution deals with robust nonparametric regression analysis when the regressors are functional random fields. More precisely, we propose a family of robust nonparametric estimators for nonparametric functional spatial regression based on the kernel method. The main results of this work are the establishment of the almost complete convergence rate of these estimators.

5.1 Introduction

The statistical problems involved in the modelization of spatial data have received an increasing interest in the literature. The infatuation for this topic is linked with many fields of applications in which the data are collected in the spatial order. The nonparametric treatment of such data is relatively recent. The first results have been obtained by Tran (1990). For relevant works on the nonparametric modelization of spatial data, see Biau and Cadre (2004), Carbon *et al.* (2007), Li *et al.* (2009) or Gheriballah *et al.* (2010). In this work, we are interested in the nonparametric spatial regression, when the covariates are of functional nature, by using a robust approach.
Currently, the progress of informatics tools and the modern technology permits the recovery of increasingly bulky data which are recorded densely over time. They are typically treated as curve or functional data. This presents the advantage to give a framework which fits better to the functional nature of the observations. For an

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overview on functional data analysis, we refer the reader to the monographs of Ramsay and Silverman (2005), Bosq (2000) for parametric models and Ferraty and Vieu (2006) for the nonparametric area . In this nonparametric context, the robust estimation of the regression function is an interesting problem in statistical inference. It is used as an alternative approach to classical methods, in particular when the data are affected by the presence of outliers. There is an extensive literature on robust estimation (see, for instance Huber (1964), Robinson (1984), Collomb and Härdle (1986), Fan *et al.* (1994) for previous results and Boente *et al.* (2009) for recent advances and references). The first results concerning the nonparametric robust estimation in functional statistic were obtained by Azzedine *et al.* (2008). They studied the almost complete convergence of robust estimators based on a kernel method, considering independent observations. Crambes *et al.* (2008) stated the convergence in L^q norm in both cases (i.i.d and strong mixing). While the asymptotic normality of these estimators is proved by Attouch *et al.* (2010).
The main aim of this contribution is to extend the results of Collomb and Härdle (1986) and Gheriballah *et al.* (2010) in the real case to the functional spatial processes. In our knowledge, this work is the first contribution on nonparametric robust regression for functional spatial variables. Specifically, we investigate almost complete convergence of the kernel estimator of the robust regression function. The interest comes mainly from the fact that an important field of application of functional statistical methods relates to the analysis of continuously indexed spatial processes.

5.2 The model

Consider $Z_{\mathbf{i}} = (X_{\mathbf{i}}, Y_{\mathbf{i}})$, $\mathbf{i} \in \mathbb{N}^N$ be a $\mathscr{F} \times \mathbb{R}$-valued measurable strictly stationary spatial process, defined on a probability space $(\Omega, \mathscr{A}, \mathbb{P})$, where $\mathscr{F}$ is a semi-metric space, d denoting the semi-metric. We assume that the process under study $(Z_{\mathbf{i}})$ is observed over a rectangular domain $\mathscr{I}_{\mathbf{n}} = \big\{\mathbf{i} = (i_1, ..., i_N) \in \mathbb{N}^N$, $1 \le i_k \le n_k, k = 1, ..., N\big\}$, $\mathbf{n} = (n_1, ..., n_N) \in \mathbb{N}^N$. A point $\mathbf{i}$ will be referred to as a *site*. We will write $\mathbf{n} \to \infty$ if $\min\{n_k\} \to \infty$ and $|\frac{n_j}{n_k}| < C$ for a constant C such that $0 < C < \infty$ for all $j,\ k$ such that $1 \le j, k \le N$. For $\mathbf{n} = (n_1, ..., n_N) \in \mathbb{N}^N$, we set $\widehat{\mathbf{n}} = n_1 \times \cdots \times n_N$. The nonparametric model studied in this paper, denoted by θ_x, is implicitly defined, for all $x \in \mathscr{F}$, as a zero with respect to (w.r.t.) $t \in \mathbb{R}$ of the equation

$$\Psi(x,t) := \mathbb{E}\left[\psi(Y_{\mathbf{i}}, t) \,|\, X_{\mathbf{i}} = x\right] = 0.$$

where ψ is a real-valued Borel function satisfying some regularity conditions to be stated below. In what follows, we suppose that, for all $x \in \mathscr{F}$, θ_x exists and is unique (see, for instance, Boente and Fraiman (1989)).

For all $(x,t) \in \mathscr{F} \times \mathbb{R}$, we propose a nonparametric estimator of $\Psi(x,t)$ given by

$$\widehat{\Psi}(x,t) := \frac{\sum_{\mathbf{i} \in \mathscr{I}_{\mathbf{n}}} K(h^{-1} d(x, X_{\mathbf{i}})) \psi(Y_{\mathbf{i}}, t)}{\sum_{\mathbf{i} \in \mathscr{I}_{\mathbf{n}}} K(h^{-1} d(x, X_{\mathbf{i}}))},$$