Abstract. The paper describes a generic tool that generates automatically natural language presentations of proofs from various automated and interactive deductive systems. Proofs from different sources are translated into a unified format and equipped with a block structure. These proofs can be easily combined. Several proof transformation procedures, based only on the analysis of structural aspects of the proofs, are available. The tool is part of the ILF system and of the ILF mail server.

1. Introduction

Cooperating systems face the user with a complex interface, consisting of a formal union of all the frequently incompatible interfaces of the singular systems that cooperate. Especially in the field of automated and interactive theorem proving – where many systems are in an experimental stage – the user has to learn not only the different input syntax but he also has to understand the output produced by the cooperating provers. These outputs are expressed in different syntax and they describe proofs in different calculi, assuming that the user is acquainted with the specific rules of inference used.

This situation makes it hard to evaluate the results of the provers in order to solve complex tasks by optimal cooperation. But even if only a single system is involved, the user might prefer to have its proofs presented in a format that can be understood by people without special knowledge on the underlying logical calculus and technical arrangements. The user who wants to understand a proof would prefer a calculus that has linear and well-structured proofs formulated in a language which is easy to understand.

In this paper, we put the problem of unified input aside and concentrate on the production of a unified natural language proof presentation for automated theorem provers. We shall describe the principles of proof
presentation that have been implemented in the system \textit{ILF} (Dahn \textit{et al.}, 1994). The \textit{ILF} system is developed at the Institute of Mathematics at the Humboldt University Berlin, supported by the Deutsche Forschungsgemeinschaft. Within \textit{ILF}, the automated theorem provers \texttt{DISCOUNT}$^1$ (Denzinger \& Pitz, 1992), \texttt{SETHEO}$^2$ (Letz \textit{et al.}, 1992), \texttt{KoMeT}$^3$ (Bibel \textit{et al.}, 1994) and \texttt{OTTER}$^4$ (McCune, 1990) cooperate with domain specific provers. \textit{ILF} itself is an interactive system.

The organization of the cooperation of the provers within \textit{ILF} is aside the focus of that paper and will be treated separately.

In early 1993 we intended to present the possibilities achieved by the cooperation of these provers to an audience without going into technical details. Therefore, we presented at a workshop in Darmstadt in March 1993 a natural language proof of Busulinis Theorem from the theory of lattice ordered groups, produced with the help of \textit{ILF}. In fact, only the proof details entered by the user had been presented, while the technical details filled in by the cooperating automated provers had been reduced to mere footnotes which indicated the authorship.

It was only legitimate to demand also a natural language proof presentation for the subproofs produced by the automated provers. Of course, these should be easy to read and fit smoothly into the general proof plan. Therefore, these proofs had to be transformed into a unified logical calculus which has well structured proofs.

Fortunately, it was possible to implement a proof presentation procedure that relied totally on the structure of the proofs and was independent of the specific syntax and set of inference rules used by a particular calculus. This structural approach is in clear contrast with the approach taken by Huang (Huang, 1994) which analyzes pattern of natural deduction inference rules in order to determine the proof presentation. It enabled a unified presentation of proofs from different provers. Moreover, it facilitates the adaptation of the proof presentation to new systems considerably.

Since the proof presentation procedures described below are to a large extent independent of the syntactic format of the formulas in the proof, they are suited equally well for sorted and unsorted first order logic and for higher order logics. Also proofs in other extensions of classical logics – like modal logics – can be presented. However, in these cases some of the tools – especially those dealing with the conversion of indirect proofs – may not be applicable.

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