THE ORIGIN OF CHAOTIC BEHAVIOUR IN THE MIRANDA-UMBRIEL 3 : 1 RESONANCES

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Abstract. We have investigated the pericentric resonances through which Miranda and Umbriel are believed to have passed when, due to tidal evolution, their orbital mean motions reached a 3 : 1 commensurability. Our investigation is based upon a perturbative treatment. The predictions of this theory are in good agreement with the results of numerical integrations concerning both the extend of the chaotic layers generated by the separatrices of the primary resonances and the location of the secondary resonances. The effect of tidal evolution is discussed on the bases of the adiabatic invariant theory and its extension to separatrix crossing. We recover qualitatively the mean features of the numerical experiments of Tittermore and Wisdom (1988-1989), Dermott et al (1988) and Malhotra and Dermott (1989).

Keywords: Resonance, chaotic motion, perturbation theory, tidal evolution, Miranda

1. Introduction

Since Voyager II has revealed that the small satellites of Uranus, Miranda, Ariel and Titania (Smith et al. 1986) show evidence of widespread melting, a considerable interest has developed in the history of the tidal evolution of those satellites.

Peale (1988) pointed out that if in the past one of these satellites had been captured into a pericentric resonance, pumping its eccentricity, and if later on the resonance was disrupted, the subsequent damping of this anomalous eccentricity could explain the melting of the satellite in the same way as the tidal heating of Io explains its volcanism (Peale et al., 1979).

Capture into resonance by tidal evolution had been investigated previously but mainly in connection with satellites of Saturn or Jupiter (Goldreich 1965, Alan 1969, Sinclair 1972, Greenberg 1973, Yoder 1973, Yoder 1979b; see also the review papers of Greenberg 1977, Peale 1976 and 1986). For these satellites, the large value of the oblateness of the planets separates the various resonances encountered near a mean-motion commensurability and a simple single resonance theory can be established (Yoder 1979a, Henrard 1982, Henrard and Lemaitre 1983). This theory explains how, when a resonance is encountered by tidal evolution, the pair of satellites can be captured in it or escape from it. It shows that when locked into resonance the eccentricities or inclinations of the satellites increase. But this theory does not provide any mechanism for subsequent escape from the resonance. Peale (1988) speculates about the action of a third satellite to disrupt the resonance.
Tittermore and Wisdom (1988) explored by numerical integration a simplified model of the 5:3 commensurability between the mean motions of Ariel and Umbriel. They found widespread chaotic behaviour and argue that because of this the predictions based upon the single resonance theory have no relevance at all. They simulated tidal evolution in a few cases and found some instances of temporary capture leading to some increase in the eccentricity. Independently, Dermott et al. (1988) investigated several resonances between Uranian satellites but especially the Miranda-Umbriel 3:1 commensurability. They observed that for low eccentricities or inclinations the single resonance theory predicts fairly well the process of capture but that when the eccentricities or inclinations grow after capture, the adjacent resonances overlap and create a chaotic domain wherein the system hops randomly from one resonance state to another one. While in this chaotic state the eccentricities or inclinations increase at a rate not too different from those predicted on the basis of the single resonance theory. Eventually the chaotic hopping was observed to lead to the "spontaneous" disruption of the resonance. Actually, this increase of the eccentricity at a rate similar to the rate predicted on the basis of the single resonance theory as the orbit follows a chaotic road is also a landmark of the simulations by Tittermore and Wisdom but was not noticed, or was found irrelevant, by the authors.

Later on, Tittermore and Wisdom (1989) investigated the 3:1 resonance in inclination of Miranda and Umbriel and found that capture followed by escape from this resonance can produce the anomalously high inclination (\(\sim 40^\circ\)) of Miranda, a result already hinted at by Dermott et al. (1988). Furthermore, Tittermore and Wisdom's investigation shows the leading role played by secondary resonance in the spontaneous disruption of the resonance. The fact that secondary resonances are an important feature of the phase space of this kind of resonance problems was already known (Henrard and Lemaître 1986, Henrard 1988). Just recently, Malhotra and Dermott (1989) have confirmed and extended the findings of Tittermore and Wisdom (1989) by clarifying the role of secondary resonances.

From all these almost simultaneous investigations emerges the following qualitative picture.

The commensurabilities of the Uranian satellites mean motions are characterized by the fact that their resonances interact strongly although they can be considered as separated for many purposes. This is in contrast to the asteroidal commensurabilities where the resonances are not separated at all and to the Saturnian or Jovian commensurabilities where the resonances can be considered as isolated for all practical purposes.

In the absence of tidal effects, the phase space still retains the imprint of the individual primary resonances, the boundaries of which are characterized by separatrices which serve as backbone to the phase space. But the strong interaction of the individual resonances replace the separatrices by a layer of chaotic motion especially when the separatrices of two individual primary resonances meet. This is also observed in the asteroidal case while in the Saturnian or Jovian cases the individual primary resonances are unaffected for all practical purposes. Inside the primary resonances we find strong secondary resonances. Their separatrices are also replaced by layers of chaotic motion. They were found also in the asteroidal case (Henrard and Lemaître 1986, Henrard and Caranicolas 1989) and, at least for 2:1 commensurability, seem more important that the primary resonances as generators of chaotic motion. The Saturnian or Jovian commensurabilities seem unaffected by secondary resonances.

The present paper is devoted to the description of this landscape in the case of the 3:1 commensurability of Miranda and Umbriel. We shall consider the planar problem and investigate the pericentric resonances. The original feature of our paper is that we shall make this description on the basis of a perturbative