This paper presents an algebraic treatment of logic, aiming to further the project of a "subjective semantics" for quantification, identity, and modality.

Semantics has largely concentrated on truth and reference, the relation of language to the world, and this is part of its usual definition. It has also been the discipline which offers theories to explain the logical behavior of various sorts of expressions traditionally studied in logic: connectors, modalities, quantifiers, abstractors, the identity predicate, and so forth. Recently a number of studies have concentrated on the relation between language and the states of mind (generally epistemic or doxastic attitudes) of its users, to provide such explanations, and the term "semantics" has also been used there. The present study will be among these. Truth and reference will be eschewed. Intuitive descriptions of the framework will be given, albeit briefly, in terms of mental operations on propositions (regarded therefore as the sort of thing which we can vary or modify in imagination). Generality will be part of the aim; neutrality with respect to certain non-classical logics will be guarded. The Appendix will show how standard semantic analyses can fit into this general framework. The algebraic (lattice-with-transformations) analysis of modality, quantification and identity given here may therefore be of some interest outside 'subjective semantics' as well.

1. INTUITIVE DESCRIPTION OF ABSTRACTION

Contemplating the proposition that Socrates is mortal, we can abstract the property of (someone's) being mortal, and generalize to produce the proposition that everyone has this property. Alternative abstractions and generalizations are possible: the proposition eventually generalized to could be that all animals have this property, or that Socrates has every property, or, somewhat further afield, that Socrates was, is, and will be mortal. There
must be many alternative abstraction operations and many associated universal quantifications.²

How does abstraction proceed? In one model of the process, the contemplated proposition is seen as complex, as having distinct ingredients (e.g. the individual concept of Socrates and general concept of mortality) and abstraction is a sort of deletion or separation. It will be a basic working hypothesis of this paper (suggested by difficulties encountered elsewhere) that propositions are au fond the only things the mind works with. There are no picture galleries of concrete and abstract individuals in addition to the propositions. (One reason: the idea of a picture without propositional content, say a picture of Socrates that is not of Socrates walking, or sitting, or standing, etc. does not make sense.) A second model of the process of abstraction utilizes the ideas of mental variation and invariance. Suppose I transform the initial proposition successively into: that Callias is mortal, that Gorgias is mortal, . . . The abstract (i.e. what is abstracted, what is 'common' to these propositions) can equally well be represented by the set of all these resultant propositions themselves. Generalization then concerns what remains invariant under these transformations.

What are such transformations like? In this example, they preserve logical structure: the inferential relations, conjunction and disjunction, perhaps much more (how much depends rather on how much logical structure the family of propositions has, a question on which our initial opinions ought to be minimal).

The example of variation I gave is a special one, and deceptively limited. It lends itself to exposition because we have the linguistic resources to describe it in simple terms. That cannot be the whole story. Contemplate a battle scene; after studying it for a moment you may conclude that all the soldiers are either fighting or wounded. You do not have a name for each one. Perhaps you went through it from left to right, saying to yourself "This one is wounded, that one is fighting, this one is wounded but still fighting, . . ." Possibly you have the linguistic resources to describe each one uniquely in qualitative terms, and if you do, perhaps you have unconsciously utilized them. But possibly not. Possibly this was a real battle scene, and the causal chains between you and it constituted genuine reference; but possibly not. Possibly it was a painting or a hallucination, or the memory of part of Trajan's Column or just something you imagined. It does not seem to matter; you are able to arrive at the general proposition