FINDING AND USING PATTERNS IN LINEAR GENERALISING PROBLEMS

ABSTRACT. Linear generalising problems are questions which require students to observe and use a linear pattern of the form $f(n) = an + b$ with $b \neq 0$. This study reports responses of students aged between 9 and 13 to these questions, documenting the mathematical models that they select, the strategies used in implementing them and the explanations they give. Substantial inconsistency of choice of model is observed; students who began a question correctly frequently adopted a simpler but incorrect model for more difficult parts of the question. Students who had undertaken a course in problem solving implicitly used a linear model more frequently and consistently and their explanations more often related the spatial patterns and the number patterns. They seemed to understand the relationship between the data and the generalising rule more completely.

INTRODUCTION

Finding and using patterns is an important strategy for mathematical problem solving. Consequently, many problem solving curriculum materials (see, for example, Burton (1984), Charles and Lester (1982), Stacey and Groves (1985), Swan (1984)) prominently feature questions which can be solved by examining special cases, organising the results systematically, finding a pattern and using it to get the answer. Lee and Wheeler (1987) labelled problems of this type “generalising problems”: the Handshakes problem is perhaps the most famous example of a quadratic generalising problem (because if $n$ people shake hands with each other there are $n(n - 1)/2$ handshakes altogether).

Although it is no longer uncommon to use generalising problems in teaching, there is little research about how students tackle them. In Routes to Roots of Algebra (Mason et al., 1985, pp. 8–11), problems of this type are used to expose four stages in “expressing generality” and reports by teachers of their use in the classroom are given. Lee and Wheeler (1987) used generalising problems involving quadratic and linear patterns in their study of generalisation and algebraic thinking of Grade 10 students. They noticed great variety of pattern perception in any one question and they found that two categories of students were successful: “those who hit upon a usable pattern perception and pushed it through [and] those that were flexible in pattern perception and could see a new pattern when one was unproductive” (p. 109). Little evidence of students’ checking their patterns is seen although Lee and Wheeler caution that even in an interview it is
often difficult to tell if checking has taken place. However, of the eight students they interviewed, only one checked that his pattern worked and he did this against the one rectangle which had inspired his formula in the first place. They conclude that “the reflex of checking the formula against the given [evidence] is not there” (pp. 112–113). Cooper (1987) and Cooper and Sakane (1986) investigated the cognitive and meta-cognitive strategies that Year 8 students employ when working (in pairs) on the quadratic generalisation problem of finding the number of chords that join \( n \) noncyclic points. In particular, they noted that when students uncovered a counter-example to their own hypothesis they tended to doubt the data that refuted it rather than acknowledge that the hypothesis was invalidated.

This study explores the responses of students aged 9 to 13 to linear generalising problems from both a technical and strategic point of view. The following questions are of particular interest:

(i) What generalisations do students make and how do they vary with increased schooling?

(ii) How do students explain the patterns they find and the generalisations they use?

(iii) How consistent are students in their choice of generalising rule (i.e. mathematical model)?)

(iv) How do the responses of students who have had some experience in generalising questions differ from inexperienced students?

LADDERS

With 8 matches, I can make a ladder with 2 rungs like this

With 11 matches, I can make this ladder with 3 rungs.

How many matches are needed to make the same sort of ladder with 4 rungs?
How many matches are needed to make a ladder with 5 rungs?
I know that it takes 335 matches to make a ladder with 111 rungs. How many matches would be needed to make a ladder with 112 rungs?
How many matches would you need to make a ladder with 20 rungs?
How many matches are needed for a ladder with 1000 rungs?

Fig. 1. Ladders.