Some physicists maintain that from a quantum-theoretic standpoint, the propositions pertaining to a physical system exhibit a non-standard logical structure, and indeed that their associated algebra is an orthomodular lattice, rather than a Boolean algebra, as in the case of classical systems. Consequently a new area of logical investigation has grown up under the name of 'quantum logic', of which one aspect is the study of the propositional logic characterised by the class of orthomodular lattices.

In recent years we have seen the development of a powerful alternative to algebraic semantics for formal systems – namely the 'possible worlds' model theory initiated by Saul Kripke. This approach was originally used to analyse modal systems, but it was soon realised that its ramifications were far wider than that. It has subsequently been applied to many other kinds of intensional logic, including tense, deontic, epistemic, intuitionist, and entailment systems, and is currently proving relevant to the study of natural languages.

The purpose of this paper is to lay a foundation for an intensional model theory for quantum logic. To do this we broaden the inquiry to encompass what we shall call Orthologic. The minimal calculus in this area is the system O, characterised by the class of ortholattices. This logic will be shown to have a semantics reminiscent of the relational structures of normal modal logic. These models are then used to establish a connection between O and the Brouwerian modal system that parallels the McKinsey-Tarski translation of intuitionist logic into S4. A discussion of filtration theory shows that O is decidable, and in the final section of the paper we extend the analysis to obtain a characterisation of quantum logic itself.

1. Syntax

The primitive symbols of our object language are (i) a denumerable collection \( \{P_i : i < \omega \} \) of propositional variables, (ii) the connectives \( \sim \) and \( \land \) of negation and conjunction, (iii) parentheses ( and ). The set \( \Phi \) of
well-formed formulae (wffs) is constructed from these in the usual way. The letters A, B, C etc. are used as metavariables ranging over $\Phi$. Parentheses may be omitted where convenient, the convention being that $\sim$ binds more strongly than $\land$. The disjunction connective $\lor$ is introduced by the definitional abbreviation $A \lor B = df \sim(A \land \sim B)$.

Our concern is to explore the relationships between two quite different ways of studying formulae. The semantical approach, to be explained in detail in the next section, has as its goal the assignment of meanings or interpretations to wffs, and the setting out of conditions under which a wff is to be true or false. The syntactical approach examines formal relationships between wffs, and focuses on the notion of consequence or derivability of formulae. In this context we can distinguish between axiomatic systems, and logics. Given a formal language, an axiom system S can be defined as an ordered pair $\langle \mathfrak{A}, \mathfrak{R} \rangle$ where $\mathfrak{A}$ is a set of wffs of the language, called axioms, and $\mathfrak{R}$ is a set of rules of inference that govern operations allowing certain formulae to be derived from others. A wff $A$ is said to be a theorem of S, written $\vdash_S A$, if there exists in S a proof of $A$ i.e. a finite sequence of wffs whose last member is $A$, and such that each member of the sequence is either an axiom, or derivable from earlier members by one of the rules in $\mathfrak{R}$.

A logic on the other hand can be thought of as a set $L$ of formulae closed under the application of certain inferential rules to its members. The members of $L$ are called L-theorems, and in this case the symbolism $\vdash_L A$ indicates merely that $A \in L$.

For example, if $S = \langle \mathfrak{A}, \mathfrak{R} \rangle$ is an axiom system, then an S-logic can be defined as any set of wffs that includes the axiom set $\mathfrak{A}$ and is closed under the rules of $\mathfrak{R}$. In general the intersection $L_S$ of all S-logics will be an S-logic, whose members are precisely those wffs for which there are proofs in $S$. This is often described by saying that $S$ is an axiomatisation of $L_S$, or that $L_S$ is generated by $S$.

Thus each axiom system has a corresponding logic (the set of its theorems) and in some formal treatments little or no distinction is made between the two. The converse however is not true. Not every logic is axiomatisable. In any semantical framework the set of wffs true in a particular model will be a logic of some kind, for which, in some cases, there may be no effectively specifiable generating procedure. A classic example is the first-order theory of the standard model of arithmetic.