PERIODIC STATES WITH FRACTIONAL FREQUENCIES IN HYBRID BISTABLE SYSTEMS

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The nonstationary operating states of bistable optical hybrid systems exposed to constant external radiation are studied. The characteristics of states with periods which are multiples of one-half the delay time of the signal in the feedback circuit are studied analytically and numerically, and it is established that they are unstable. It is shown that near certain resonance frequencies the output signal of the system is highly sensitive to periodic or noise perturbations.

The action of stationary external radiation on nonlinear optical systems with a delay, giving rise under certain conditions to the appearance of periodic and stochastic pulsations of the intensity, is being widely studied both theoretically and experimentally. Here hybrid bistable optical systems (BOS) are the most convenient objects for analysis and experiments. For such BOS, in the case of long delay times in the feedback circuit \( \tau_d \), it was shown in [1] that there exist periodic (with periods which are multiples of \( \tau_d \)) and stochastic states. Further theoretical and experimental studies [2, 3] have shown that the appearance of periodic states with successive period doubling (\( 2\tau, 4\tau_d, 8\tau_d, \ldots \)) in the course of a transition from the stationary state to the chaos obeys Feigenbaum's similarity law [4].

In purely optical bistable systems, for which Feigenbaum's scenarios were predicted in [7], the mechanism of the transition to chaos can be different (see, for example, [8]). However, even in hybrid BOS a number of important questions remain unresolved. This concerns, in particular, the periodic states found in [9] with "fractional periods" (\( 2\tau_d/3, 2\tau_d/5, 2\tau_d/7, \ldots \)). Ikeda et al. [9] concluded that they are stable and with fixed parameters of the system several such states coexist. Closely related with these states is the interpretation, given in [10], of the chaos and "windows of periodicity" as a nonlinear interaction of the "characteristic modes" of the corresponding linearized system. This approach is difficult to reconcile with Feigenbaum's model [4].

In this work the characteristics of states with "fractional periods" are studied in greater detail, both analytically and numerically, and it is established that they are unstable. It is also shown that the output signal of the hybrid BOS is highly sensitive to periodic or noise actions in the vicinity of the frequencies of the "characteristic modes."

The differential-difference equation describing the states of the hybrid BOS with no cavity [11] has the form

\[
\tau_r \frac{dx(i)}{dt} + x(i) = f(x(t-\tau_d));
\]

\[
f(x(t)) = PT(x) = P \sin^2(\varphi + x(t)),
\]

where \( \tau_r \) is the relaxation time of the electric signal \( x(t) \) in the feedback circuit and \( T(x) \) is the dimensionless intensity coefficient of transmission of the external radiation \( P \) in the feedback circuit. The reduced form of \( T(x) \) to a good approximation corresponds to electro-optical modulators, and in addition \( \varphi \) is the constant electric bias on the modulator.

*The transition to developed chaos also includes a reversed sequence of bifurcations of quasi-periodic solutions with halving of the period of the harmonic component [5, 6].

Equation (1) can have several stationary branches with different regions of stability of stationary states. We shall study the possibility of the appearance of periodic solutions near the stationary solutions. We introduce the deviation from the stationary solution on the fixed branch $\Phi(t) = x(t) - x_{st}$. Then for $\Phi(t)$ from (1) we obtain

$$t_t (d\Phi/dt) + \Phi(t) = g(\Phi(t - t_d)).$$

We shall represent $g$ in the form of a series in powers of $\Phi(t)$:

$$g(\Phi(t)) = \sum_{n=1}^{\infty} \frac{f'(x_{st})}{n!} \Phi^n(t) = \sum_{n=1}^{\infty} g_n \Phi^n(t).$$

For BOS with the transmission function (2) the first coefficients in the expansion have the form

$$g_1 = P \sin 2(q + x_{st}), \quad g_2 = P \cos 2(q + x_{st}),$$

$$g_3 = -(2/3) P \sin 2(q + x_{st}).$$

The stability of the stationary solution $x_{st}$ is determined by the characteristic equation

$$t_t \gamma + 1 - g_1 \exp(-\gamma t_d) = 0,$$

obtained by linearizing (3). Equation (6) has an unbounded set of solutions (see, for example, [12]). The real part of each root $\text{Re} \gamma = \gamma_n$ characterizes the rate of decay of the corresponding mode of the linearized system, while the imaginary part characterizes its frequency. When $\text{Re} \gamma > 0$, the stationary state is unstable relative to the excitation of this mode.

Separating the real and imaginary parts of (6) it is possible to determine the bifurcation points ($\text{Re} \gamma = 0$, $\text{Im} \gamma = \omega_n$) and the frequencies of the linearized problem $\omega_n$:

$$\cos(\omega_n t_d) - \omega_n t \sin(\omega_n t_d) = g_{10};$$

$$\sin(\omega_n t_d) + \omega_n t \cos(\omega_n t_d) = 0.$$

Here $g_{10}$ is the value of $g_1$ at the point of bifurcation (on the boundary of the region of stability of the stationary solution), determined from (7a) after the characteristic frequency is found from (7b). The set of characteristic frequencies corresponds to the points of intersection of the family of tangents with the straight line

$$\tan(\omega_n t_d) = -\frac{1}{t_t + t_d} (\omega_n t_d), \quad n = 0, \pm 1, \pm 2, \ldots$$

In what follows we shall ignore all frequencies with even numbers, since they correspond not to the boundary of stability, but rather the boundary of existence of the stationary solution. The deviation from each bifurcation point can be characterized by the deviation of different interrelated bifurcation parameters from the corresponding values at the stability boundary. For example $P = P_0^n (1 + \mu^n)$ or $g_1 = g_{10} + \delta g^n$. At the same time, at the bifurcation point $P = P_0^n, \quad g_1 = g_{10}, \quad \mu^n = 0, \quad \delta g^n = 0$. It is convenient to express stability indices and corrections to the frequencies in terms of $\delta g^n$. Solving Eqs. (7) near each bifurcation point, we find the corresponding stability characteristics $\gamma_n$ and corrections to the frequencies $\delta \omega_n (\omega = \omega_n + \delta \omega_n)$:

$$P_n = g^n \cos(\omega_n t_d) \overline{t_t + t_d}, \quad \delta \omega_n = - g^n \sin(\omega_n t_d) \overline{t_t + t_d}.$$

The sign of $P_n$ is determined by the sign of $\delta g^n$, since $\cos(\omega_n t_d) < 0$ for odd $n$ (8). Specific calculations for the transmission function (2) show that the increase in the intensity $P$ corresponds to the negative shift ($\delta g^n < 0$) from the bifurcation point, and a decrease in $P$ corresponds to a positive shift ($\delta g^n > 0$); i.e., according to (9), the stationary state becomes unstable relative to the excitation of this type of oscillation accompanying a transition through $g_{10}^{(n)}$ with increasing intensity of the external signal. The boundary of stability of...