Abstract. We give a condensed survey of recent research on generalized quantifiers in logic, linguistics and computer science, under the following headings: Logical definability and expressive power, Polyadic quantifiers and linguistic definability, Weak semantics and axiomatizability, Computational semantics, Quantifiers in dynamic settings, Quantifiers and modal logic, Proof theory of generalized quantifiers.

1. Introduction

The study of generalized quantifiers is by now an old and respectable field of logic. With the pioneering work of Mostowski and Lindström in the fifties and sixties, quantifiers became a major tool in the model theory for logics extending first-order logic — many of these being representable as first-order logic with added quantifiers. Apart from general structure theorems on how various general properties are distributed in this class of logics (most famous of these is still Lindström's theorem on the properties which characterize first-order logic), particular logics were examined in detail w.r.t. their model theoretic properties and their comparative expressive power, as well as the behaviour of theories expressed within these logics. Though some of the extensions transcend first-order models (e.g. logics with measure-theoretic or probabilistic quantifiers), this work, which reached its peak in the late seventies and early eighties — witness the book *Model-Theoretic Logics* edited by Barwise and Feferman — is squarely situated within clas-
sical model theory, with mathematics as its main source of inspiration and set theory as its basic framework.

In the beginning eighties the study of quantifiers received an impetus from a quite different direction, when it was realized (by Barwise and Cooper, Keenan and Stavi, and others) that determiners and noun phrases, which abound in most natural languages, were interpreted in Montague style semantics by means of generalized quantifiers. This brought parts of the established model theory of quantifiers to bear on linguistics, but it also brought new logical questions about quantifiers, motivated by the linguistic perspective and by particular constraints inherent in natural languages (such as conservativity, or the use of finite or at least 'small' models).

Research on quantifiers stemming directly from the original waves of inspiration (Lindström’s theorem and Montague semantics, respectively) has perhaps had its hey-day, but the field does not show signs of exhaustion. On the contrary, a lot of work on quantifiers is going on, addressing not only 'classical' issues, but also extending them in new directions, charting new territories and establishing sometimes surprising connections with other fields. One such connection is with finite model theory as used in descriptive complexity theory in computer science. Another is with recent developments in modal logic. Both will be elaborated on below.

Our aim here is to indicate the direction of some of this recent research, sketching a few major research areas and research problems, in a way that hopefully may be useful both for the practitioner in the field and for the interested logician/linguist, and also for students looking around for something to set their teeth in. At least, that is our intention. Moreover, through this unified presentation, we hope to illustrate, and to encourage the current confluence and interaction of more mathematical and more linguistic research lines in this area. After some background, the material is presented under eight distinct headings. This is for ease of exposition, but it will become clear that much of the work is interconnected and some of it belongs under more than one heading.

Thus, this paper is not a scholarly survey but rather a condensed 'state of the art' document. Extensive surveys of generalized quantifier theory and its uses in various fields already exist (cf., in addition to the volume edited by Barwise and Feferman, van Bentham 1986, Westerståhl 1989, Krynicki, M. Mostowski and Szcerba 1994, Keenan and Westerståhl 1994), though, to our knowledge, none that covers all the aspects of quantifiers signalled here.