HEURISTIC ALGORITHMS FOR DELIVERED PRICE SPATIALLY COMPETITIVE NETWORK FACILITY LOCATION PROBLEMS

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Abstract

We review previous formulations of models for locating a firm's production facilities while simultaneously determining production levels at those facilities so as to maximize the firm's profit. We enhance these formulations by adding explicit variables to represent the firm's shipping activities and discuss the implications of this revised approach. In these formulations, existing firms, as well as new entrants, are assumed to act in accordance with an appropriate model of spatial equilibrium. The firm locating new production facilities is assumed to be a large manufacturer entering an industry composed of a large number of small firms. Our previously reported proof of existence of a solution to the combined location-equilibrium problem is briefly reviewed. A heuristic algorithm based on sensitivity analysis methods which presume the existence of a solution and which locally approximate price changes as linear functions of production perturbations resulting from newly established facilities is presented. We provide several numerical tests to illustrate the contrasting locational solutions which this paper's revised delivered price formulation generates relative to those of previous formulations. An exact, although computationally burdensome, method is also presented and employed to check the reliability of the heuristic algorithm.

1. Introduction and notation

In this paper, we present and numerically test an extension of a spatially competitive facility location heuristic algorithm originally formulated by Tobin and Friesz [26], and applied to some small "toy" networks by Friesz, Miller and Tobin [6]. The extended heuristic, which is the subject of this paper and which we refer to as the delivered price spatially competitive facility location model, was first described by Friesz, Tobin and Miller [8], who analyzed the existence of solutions when certain regularity conditions are satisfied. The present paper reports the first numerical tests of this heuristic, with the aim of empirically depicting the effect of ownership of transportation infrastructure on the entering firm's strategy. Such numerical experiments are required since the model's solutions, due to its daunting complexity, have not been successfully analyzed in closed form to date.
The delivered price spatially competitive facility location model locates a firm's production facilities and determines production levels at those facilities so as to maximize the firm's profit, taking into account the effect of both the firm's production and shipping activities on market prices. We assume that competition exists among the firms and, in particular, between the locating (i.e. entering) firm and those already in place. This locating firm is further assumed to be a large manufacturer entering an industry composed of small firms. Thus, it acts as the leader of the industry.

We restrict the firm's possible locations to a subset of nodes of a graph representing the transportation network, and require that movements of the commodity produced and resulting prices correspond to a competitive equilibrium. The site location restriction is intended to mimic the typical decision making process of many firms who frequently limit a location search to an initially determined set of potential locations. As in our original formulation, we assume that the rest of the market can be represented as a network spatial price equilibrium (SPE). We extend this first model by introducing "transport" variables which explicitly represent the firm's shipments of its newly located production to demand markets. Previously, we assumed that the entering firm, like all other firms and consumers, utilized the transport services available from the transport industry specified in the SPE model. Thus, in our new heuristic, the entering firm recognizes that it can influence market prices by its integrated shipping and production decisions, whereas in the original formulation, the entering firm could only impact prices through its production decisions. This distinction yields the potential for increased relative market power and profits for the entering firm in the new model.

To differentiate these two models further, one can also distinguish our revised heuristic algorithm from our original formulation by noting that it (the new heuristic) represents a "delivered price" type model, while the original model represents an FOB plant type model. For purposes of clarity during the balance of the paper, we will denote these two models as "delivered price" and "FOB" models, respectively.

We present numerical tests of both our revised and original heuristic, and compare the revised heuristic's solutions to those previously obtained by our original model over identical networks. The contrasting solutions of the two models illustrate that ownership of transportation services can have a significant impact upon the entering firm's location decisions and profits. We also, however, utilize a numerical example to illustrate that our revised heuristic obtains the identical solution as our original model, in the case where the firm's private transport costs are so high or uneconomic as to have a negative impact on its profits.

We have previously offered an original proof of existence of a solution to this model formulation [8]. In that same paper, we also showed that the regularity conditions used to establish existence of a solution to the combined location–equilibrium problem may be employed to establish existence of the appropriate derivatives needed to apply the perturbation heuristic. Thus, these issues will receive only brief reference and summary in this paper.