Frustrated Percolation, Spin Glasses and Glasses (*).

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Summary. — The static and dynamic properties of the frustrated percolation model are investigated. This model, which contains frustration as an essential ingredient, exhibits two transitions: a percolation transition at a temperature $T_p$ with critical exponents of the ferromagnetic ($s = 1/2$)-state Potts model, and a second transition at a lower temperature $T_g$ in the same universality class of the Ising spin glass model. Above $T_p$ the time-dependent autocorrelation function is characterized by a single exponential, while for $T_p > T > T_g$ preliminary numerical results show a broad shoulder or plateau typical of a structural glass transition. Below $T_g$ the system is in glassy state with an infinitely long relaxation time.

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1. – Introduction.

One of the features of many complex systems is the appearance of many minima in the free energy, which allows the system to exist in one of many different states for a given set of external parameters. The corresponding rough landscape in configurational space induces a complex dynamical behaviour, characterized by relaxation phenomena occurring on many time scales. At low temperatures, the system may become trapped in one of the minima and exhibit non-ergodic behaviour. One mechanism leading to a landscape with many minima is based on the concept of frustration. In spin glasses [1,2] (SG), frustration occurs when the ferromagnetic and antiferromagnetic interactions are distributed in such a way that not all the spins can satisfy all the interactions simultaneously. In glass-forming systems[3] without underlying crystalline order, frustration arises because the molecules are intrinsically unable to form close-packed configurations at low temperature or high

density, while for systems with underlying crystalline order, frustration arises when the local arrangement of molecules kinetically prevents all the molecules from reaching the crystalline state.

The spin glass transition is known to be characterized by a diverging correlation length associated with the square of the spin-spin correlation function, and consequently exhibits a strong divergence in the non-linear susceptibility. However, it is currently a matter of great debate as to whether or not the glass transition in structural glasses also exhibits a diverging length[4]. Nevertheless, both spin glasses and glass-forming liquids exhibit complex dynamical behaviour at low temperatures (and also high densities in the case of liquids), well before the transition is reached. At high temperatures, typical autocorrelation functions \( f(t) \) are found to decay in time with a single exponential in both systems. As the temperature is lowered, a new behaviour sets in around a temperature \( T_c > T_g \), which in liquids is characterized by a fast decay (often called the \( \beta \)-relaxation regime) followed by a slow decay (often called the \( \alpha \)-relaxation regime), resulting in a broad shoulder or plateau in \( f(t) \) when plotted as a function of log \( t \) (fig. 1a)). In spin glasses, the first relaxation is extremely fast\[5, 6\], being of the order of the time for an individual spin to flip, and also the plateau preceding the second cooperative relaxation process is less pronounced. In both systems, the long-time behaviour is often well approximated by a Kohlrausch-Williams-Watts (KWW) function, also known as a stretched exponential:

\[
f(t) \sim \exp \left[ -\left( t/\tau \right)^{\beta(T)} \right],
\]

where \( \tau \) is the relaxation time that diverges as \( T \) approaches \( T_g \) or \( T_{SG} \) from above. In spin glasses, the exponent \( \beta(T) \) is found to vary from 1 to 0.3 as the temperature is lowered from \( T_c \) to \( T_{SG} \).

To explain the complex behaviour just described is one of the key requirements of a theory of the glass transition. Mode-coupling theory\[7\] of simple, dense fluids, in both its original and modified version, predicts much of the dynamical behaviour observed in supercooled liquids, and for this reason is currently a popular theory of glass-forming liquids. However, to gain more insight into the physics of frustrated systems in general, it would be of great utility to have a simple lattice model containing the basic ingredients essential for reproducing the phenomenology of the glass transition.

In this paper we present a new lattice gas model that contains frustration as an essential ingredient. Since percolation[8] plays an important role in determining the critical behaviour of this model, the model has been called «frustrated percolation»[9]. In fact, it can be viewed as a percolation model (in the two variants of site and bond percolation) in a frustrated medium. Both the static and the dynamic properties of the model exhibit complex behaviour with features in common with both structural glasses and spin glasses. This may suggest that, just as the Ising model and its lattice gas version provided insight into the physics of critical phenomena near second-order phase transitions, as well as related the liquid-gas transition to the paramagnetic-ferromagnetic transition, so the frustrated lattice gas model presented here may more closely relate the glass transition in liquids to the spin glass transition.

We begin by defining the concept of frustration in spin glasses and glasses in sect. 2. In sect. 3, the site-frustrated percolation (SFP) model is introduced to