NOTE

ANALYTICAL CONSIDERATIONS ON A NEW CLASS OF FUNCTIONS FOR THE MATHEMATICAL DESCRIPTION OF THE HUMAN ERYTHROCYTE’S PROFILE DURING THE OSMOTIC, "DISK-SPHERE" TRANSITION

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An analytical procedure is used to derive a class of functions for the shapes of the cross section of the human red cell during its osmotic swelling.

1. Introduction.
Several functions concerning the profile of the meridian section of the human red cell (HRC) have been introduced for the analytical description of cell’s morphology during its osmotic swelling. With regard to the first quadrant of a plane (Oxy) the commonest functions are the following, put forward by Cerny (1971), Canham (1970), Evans and Fung (1972) respectively:

\[ y_1 = \frac{\sqrt{-(x^2 + a^2) + \sqrt{4a^2x^2 + m^4}}}{\sqrt{4a^2x^2 + m^4}} \]

\[ y_{II} = B \sqrt{-(x^2 + a^2) + \sqrt{4a^2x^2 + m^4}} \]

\[ y_{III} = (a + bx^2 + cx^4) \sqrt{R^2 - x^2}. \]

Clearly it is possible to propose many other functions for the same
purpose, as, for example,

\[ y_{1N} = \frac{m\sqrt{R^2 - x^2}}{N\sqrt{R^2 - x^2}} \]

proposed by Martino \textit{et al.} (1977), or the models suggested by Reeves and Whitmore (1977).

The problem is to find a function having three properties simultaneously: (1) simplicity, (2) morphogenetic power, and (3) ease of employment; and the purpose of the present paper is to suggest a class of such functions, consistent with the above-mentioned properties. We shall denote this class as the class of the \( y_N(x) \) functions.

2. Analysis. In order to determine a parametric function, \( y(x) \), suitable for the analytical description of the profile of the meridian section of HRC during the osmotic, "disk-sphere" transition, we shall start from \( y'(x) \) rather than from \( y(x) \). In the following it will be assumed, obviously, that

\[ y'(0) = 0, \]

\[ y'(R) \to -\infty, \]

\[ \int_R^0 y'(x)dx = P, \quad y(R) = 0, \]

where \( R \) is the "equatorial" radius and \( P \) is the "polar" radius (see Figure 1).

Clearly we have:

\[ y(x) \overset{\text{def}}{=} - \int_x^R y'(t)dt \]

Let us write: \( y'(x) = g(x) \). Therefore, in accordance with the above definition,

\[ y(x) = \int_x^R g(t)dt, \]

where the domain of the \( y(x) \) is \( 0 \leq x \leq R \). We have \( y(0) = P \) and \( y(R) = 0 \). In