ON SEMIGROUPS WITH MINIMAL OR MAXIMAL CONDITION
ON LEFT CONGRUENCES

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Introduction. The semigroups in the title are called here the DCC- or, respectively, ACC-semigroups (DCC = descending chain condition, ACC = ascending chain condition, as usual). In [3], [4], E. Hotzel initiated the study of such semigroups and had obtained many interesting results. In particular, any DCC-semigroup and any weak periodic ACC-semigroup has only a finite number of left ideals ([3], Theorem 3 and [4], Theorem 2.1). Further, a DCC-semigroup (resp., a weak periodic ACC-semigroup) is finite if all of its subgroup are finite ([3], Theorem 9 and [4], Theorem 2.3).

The purpose of this work is to discover the structure of the O-simple, Clifford, inverse and commutative semigroups satisfying ACC or DCC. Recall that commutative ACC-semigroups were characterized by Budach [1], namely, these semigroups are exactly the finitely generated semigroups. (The sufficiency has been proved previously by L. Rédei [6].)

Section 1 of this work has an auxiliary character. We provide in it the necessary definitions and notations and several results on arbitrary semigroups with ACC or DCC.

In Section 2, we describe the O-simple ACC- or DCC-semigroups and give examples illustrating the main proposition of this Section.

In Section 3, we prove the finiteness of a Clifford semigroup satisfying ACC or DCC on two-sided congruences and having no infinite subgroups.
Finally, in Section 4, we are concerned with the commutative and inverse semigroups. We prove that a commutative semigroup is a DCC-semigroup if and only if it has a principal series and satisfies DCC on subgroups. Further, an inverse semigroup is a DCC- or ACC-semigroup if and only if it has only a finite number of idempotents and satisfies DCC or ACC on subgroups.

Many of results of this article were announced by the author in [7].

1. The basic definitions and notations.

Preliminary results

We follow the terminology and notation of the monograph [2]. Moreover, a semigroup which is a union of groups is called Clifford semigroup. A semilattice is a commutative idempotent semigroup. It becomes a partially ordered set if we put $e \leq f \iff ef = e$.

The symbol $L(S)$ is used in this work for the lattice of left congruences of a given semigroup $S$. Further, $\Delta_g = \{(s,s) \mid s \in S\}$ is the equality relation on $S$. The partially ordered set $Y$ is said to be an antichain if the elements of $Y$ are pairwise incomparable.

As usual, we use $\mathcal{L}$ and $\mathcal{H}$ for the Green relations on a given semigroup. The symbol $M^\circ(G,I,\Lambda,P)$ means the Rees matrix semigroup over $G^\circ$, a group with zero, with sandwich-matrix $P$ ($I$ and $\Lambda$ are respectively the set of columns and rows of $P$); the symbol $M(G,I,\Lambda,P)$ means the Rees matrix semigroup without zero. In the semigroup $M^\circ(G,I,\Lambda,P)$ for $i \in I$ we put $i(\Lambda) = \{ \lambda \in \Lambda \mid p_{\lambda i} \neq 0 \}$. For $M \subseteq G$, $i \in I$, $\lambda \in \Lambda$ let $M_{i,\lambda} = \{(g)_{i,\lambda} \mid g \in M\}$. For a set $M$ let $|M|$ is the power of $M$. Finally, we emphasize that the symbol $\subset$ means here only strong inclusion.

We now begin to consider the semigroups with chain conditions. As was noted by E. Hotzel, the distributivity