

# Did Poincaré Say “Set Theory Is a Disease”?

Jeremy Gray

One of the most colourful quotes in all the history of mathematics is attributed to Poincaré: “Later generations will regard *Mengenlehre* as a disease from which one has recovered.” The quotation is perhaps a little spoiled by the intrusion of the German word for set theory, but doesn’t that give it just the aura of verisimilitude required for maximum conviction? How fortunate to have an illustration of those vigorous expressions of opinion we all know mathematicians make in private but seldom confide to the page; how fortunate too that Poincaré chose to lambast a subject we and all our students know, the hapless core of modern mathematics. Had he chosen, shall we say, the theory of higher order Bessel functions, the quote would surely be less well known. I wish, however, to suggest that Poincaré never made such a remark. Since the quote is used to buttress an argument that Poincaré was strongly opposed to the study of the theory of sets, if the quote goes down, then Poincaré’s position can be seen to be richer, more interesting, and more profound, if less colourful. (Any reader who can find reliable testimony that Poincaré did make this remark is cordially invited to let me know so that the source can be published.)

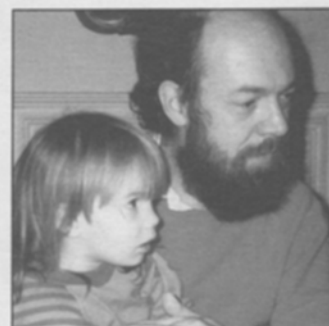
I became suspicious of the purported quotation when I realised that it was far cruder in its attitude toward the study of set theory than any other remark I could find by Poincaré. My suspicion deepened on seeing that none of the most scholarly recent writing on Poincaré carries the remark. Gregory Moore [8], and Jean Cassinet and Michel Guillemot [1] between them devote many pages to how Poincaré’s views evolved during the 1900s, but they never allude to this remark. J. W. Dauben’s equally reliable and thorough study of Cantor [2] also neglects to carry it. This suggests that none of these authors found it in an authentic text of Poincaré.

It is, however, quoted in a number of places. Morris Kline’s [7] has it on page 1003. Amongst popular sources I cite Martin Gardner’s [5], page 27. We can start with Kline’s account, which is in a well-respected scholarly source. It runs:

Poincaré<sup>37</sup> remarked critically, “But it has happened that we have encountered certain paradoxes, certain apparent contradictions which would have pleased Zeno of Elea and the school of Megara. . . . I think for my part, and I am not the only one, that the important point is never to introduce objects that one cannot define completely in a finite number of words.” He refers to set theory as an interesting “pathological case.” He also predicted (in the same article) that “Later generations will regard [Cantor’s] *Mengenlehre* as a disease from which one has recovered.”

If you consult any of the sources that Kline gives in footnote 37, the *Proceedings of the Fourth International*

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with daughter Martha



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Henri Poincaré

*Congress of Mathematicians*, Rome, 1908, 167–182; the *Bulletin des sciences mathématiques*, (ser. 2), vol. 32, 1908, 168–190; extract in *Oeuvres*, 5, 19–23, you are referred to Poincaré's essay *L'Avenir des Mathématiques*. However, you will not find the remark. In the case of the *Oeuvres* none of the passage appears, because the editors chose to reproduce, as they say, only a part of Poincaré's essay. The other two sources, which are identical, only carry the passage about "certain paradoxes" pleasing to Zeno, the suggested remedy, and the suggestion that set theory is a pathological case. In particular, the remark we seek is not in the article from which the other remarks are taken.

A faulty attribution of a source does not mean that Poincaré did not pass his memorable judgement. Nonetheless, it is worth speculating on where Kline may have taken his information from. His account closely follows that of E. T. Bell, *Men of Mathematics*, first published in 1937. On page 558 of the Fireside reprint, 1965, we find essentially the same quotation down to the remark about using only a finite number of words. Bell then continued:

Whatever be the cure adopted, we may promise ourselves the joy of a physician called in to treat a beautiful pathologic case.

A few years later Poincaré's interest in pathology for its own sake had abated somewhat. At the International Mathematical Congress of 1908 in Rome, the satiated physician delivered himself of this prognosis: "Later generations will regard *Mengenlehre* as a disease from which one has recovered."

The English of these two versions of Poincaré's text differs up to and including "a finite number of words." Then comes the remark about the pathological case. So far, both are fair translations of Poincaré's essay, which suggests that both Kline and Bell had independently consulted it. But the English of the quote we seek is given by Kline exactly as it is found in Bell, which suggests that this is (one of) Kline's sources. However, there are things wrong with the story as it is found in Bell. Although he quoted correctly from Poincaré's essay, *L'Avenir des Mathématiques*, he attributed it to 1905. However, as the French *Bulletin* and the Italian *Rendiconti del Circolo Matematico di Palermo*, 26, 1908, 152–168, where the essay was also printed, make clear, the essay was circulated at the 1908 International Congress, so it represents Poincaré's views of that date. Bell has conjured up a shift in Poincaré's thought. Perhaps Kline, on seeing that *L'Avenir* in fact dates from 1908, tidied up that error in Bell, only to make another, namely that the famous remark is to be found in the text of *L'Avenir*.

Indeed, Bell does suggest that the remark was part of the speech. But he does not explicitly say so, which leaves open the possibility that it could have been a throwaway remark, either made at the time and then suppressed or else made in conversation. It is unlikely to have been made at the time, for as footnotes in the *Bulletin* and the *Rendiconti* make clear, Poincaré did not in fact give the speech himself. He was temporarily ill, and Darboux read it for him. I must assume that Darboux stuck to the text, as it was circulated in advance in the form of a pamphlet published by the Circolo di Palermo. Poincaré was associated with them at the time; at the Rome conference he adjudicated their prize competition, awarding a medal to Severi. Were it not too fanciful, one might imagine Darboux looking up and saying something like "Of course, what Poincaré really thinks is . . ." Perhaps, but none of the sources say that. Nor do they say that informally, in conversation, Poincaré made the remark. Nor did Bell attend the conference and catch the remark himself, for his name does not appear in the *Elenco dei Congressisti* given on page 12 of the *Atti del congresso internazionale dei matematici*, 1908, vol. 1.

However, there is no need for such uncharitable speculation, for there is a source earlier than Bell from which he may well have taken the story, and which enables us to come close to proving a negative. This is an essay by James Pierpont called *Mathematical rigor, past and present*, given as an invited address at the American Mathematical Society's meeting in Nashville, 1928, and printed in the *Bulletin of the AMS*. Bell may even have heard the address, but we can be sure he read it, even though he does not cite it. He used some of Pierpont's phrases and also the same quotes from Brouwer that Pierpont gave, as well as following the main line of Pierpont's argument. Purists may