Abstract. We compare two conditional logics for defeasible inference: Alchourrón’s defeasible logic DFT and Boutilier’s CO logic. The two logics share the distinguishable characteristics common to most logics for defeasible inference. Namely, their conditional connective defeat the rules of Modus Ponens, Strengthening the Antecedent, Transitivity, and Contraposition. Although both logics have possible worlds semantics, Boutilier’s is relational while Alchourrón’s is non-relational. In this note we reveal the connection between the two, concluding that the conditional sentences validated by both logics are precisely the same.

1 Introduction

In this note we compare two modal conditional logics for defeasible inference: Alchourrón’s DFT [2] and Boutilier’s CO [3]. Both are conditional logics with possible worlds semantics. Boutilier gives a relational semantics, requiring a reflexive transitive and totally connected binary relation between worlds. He regards the accessibility relation as a plausibility ordering of possible worlds. In contrast, Alchourrón gives non-relational semantics, based on a selection function $Ch$ that picks all presuppositions for a given proposition. Both logics have an axiomatic presentation, and are sound and complete. In this note we investigate their connection. Both conditional constructions share the distinguishable characteristics with respect to the conditional connective common to most logics for defeasible inference. Namely, they defeat the rules of Modus Ponens, Strengthening the antecedent, Transitivity, and Contraposition.

2 Alchourrón’s DFT Logic

Alchourrón’s modal conditional logic is based on a propositional language $L_{CPL}$ (over variables $P$, the set of atomic variables in the language) augmented with an S5-necessity operator $\Box$ and a revision operator $f$. The resulting modal language is denoted by $L_M$. The possibility operator $\Diamond$ defined in terms of $\Box$ as usual: $\Diamond A \equiv_{df} \neg\Box \neg A$. Alchourrón bases his construction on the very idea that in a defeasible conditional the antecedent is a contributory condition of its consequent (as opposed to be a sufficient condition for the consequent). Hence, he
defines a defeasible conditional \( A \succ_{dpt} B \) meaning that the antecedent \( A \) jointly with the set of assumptions that comes with it is a sufficient condition for the consequent \( B \). In order to represent in the object language the joint assertion of the proposition expressed by a sentence \( A \) and the set of assumptions (or presuppositions) that comes with it he uses a revision operator \( f \). Let \( A_1, \ldots, A_n \) the set of assumptions associated with \( A \), then \( fA \) stands for the joint assertion (conjunction) of \( A \) with all the \( A_i \) (for all \( 1 \leq i \leq n \)), where \( A \) is always one of the conjuncts of \( fA \). Four constraints are imposed to the revision operator \( f \):

\begin{align*}
\textbf{f.1} & \quad (fA \supset A) \quad \text{(Expansion)} \\
\textbf{f.2} & \quad (A \leftrightarrow B) \supset fA \leftrightarrow fB \quad \text{(Extensionality)} \\
\textbf{f.3} & \quad \Diamond A \supset \Diamond fA \quad \text{(Limit Expansion)} \\
\textbf{f.4} & \quad [f(A \lor B) \leftrightarrow fA] \lor [f(A \lor B) \leftrightarrow fB] \lor [f(A \lor B) \leftrightarrow (fA \lor fB)] \\
& \quad \text{(Hierarchical Ordering)}
\end{align*}

Condition \( \textbf{f.1} \) is quite natural since \( fA \) stands for conjunction of \( A \) and its presuppositions. The second asserts that equivalent sentences have equivalent presuppositions. \( \textbf{f.3} \) insures the existence of consistent presuppositions for any sentence that is not a contradiction. We will see below that the formulation of condition \( \textbf{f.3} \) carries some consequences that we analyze in semantic terms. Finally, \( \textbf{f.4} \) asserts that the presuppositions of a disjunction are either the presuppositions of one of the disjuncts, or else the disjunction of the presuppositions of each of the disjuncts. Alchourrón defines a defeasible conditional \( A \succ_{dpt} B \) meaning that the antecedent \( A \) jointly with the set of assumptions that comes with it is a sufficient condition for the consequent \( B \). To reflect his intuition, Alchourrón adopts the following definition due to Lennart Åquist:

\[
A \succ_{dpt} B \equiv_{df} \Box (fA \supset B)
\]

Alchourrón gives a formal semantic interpretation of the object language \( L_M \) based on standard non-relational S5-models. He defines a model for \( L_M \) as \( M_{dpt} = (W, Ch, \| \|) \) where \( W \) is a non-empty set of possible worlds, \( \| \| \) is the valuation function (\( \| \| \) maps \( P \) into \( 2^W \)), and \( Ch \) is a selection function such that for each sentence \( A \) of \( L_M \) \( Ch(A) \subseteq W \). That is, \( Ch(A) \) is the proposition of the joint assertion of \( A \) and its assumptions, i.e., the worlds in which \( fA \) are true. Hence, Alchourrón defines:

\[
\| fA \| = Ch(A)
\]

The function \( Ch \) satisfies the following four constraints, in exact correspondence to the four properties of the revision operator \( f \).

\begin{align*}
\textbf{Ch.1} & \quad Ch(A) \subseteq \| A \| \\
\textbf{Ch.2} & \quad \text{If } \| A \| = \| B \| \text{ then } Ch(A) = Ch(B)
\end{align*}

\(^1\) Alchourrón defines the selection function \( Ch \) as \( Ch^a \) meaning that the selection is indexed by the particular preferences of an individual \( a \) (as opposed to be a universal selection function for every individual). For the purposes of this note, the distinction is not relevant; moreover, wherever we write \( Ch \) it could be read as \( Ch^a \).