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ORIGINAL ARTICLE

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Optimal scheduling of a two-stage hybrid flow shop

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Abstract We present an exact branch-and-bound algorithm for the two-stage hybrid flow shop problem with multiple identical machines in each stage. The objective is to schedule a set of jobs so as to minimize the makespan. This is the first exact procedure which has been specifically designed for this strongly $\mathcal{NP}$-hard problem. Among other features, our algorithm is based on the exact solution of identical parallel machine scheduling problems with heads and tails. We report the results of extensive computational experiments on instances which show that the proposed algorithm solves large-scale instances in moderate CPU time.

Keywords Deterministic scheduling · Hybrid flow shop · Branch-and-bound method

1 Introduction

The two-stage hybrid flow shop scheduling problem may be formulated as follows. A set J of n jobs has to be scheduled in a manufacturing system with two stages (machining centers) Z_1 and Z_2. Each stage Z_i $(i = 1, 2)$ has m_i identical machines in parallel. Each job j $(j = 1, \ldots, n)$ has to be processed first for a_j units of time by one machine of Z_1, and then for b_j units of time by one machine of Z_2. These operations must be processed without preemption. Moreover, a job cannot be processed by more than one machine at the same time and each machine processes at most one job at one time. All processing times are assumed to be deterministic and integer and all machines are ready from time zero onwards. The objective is to construct a schedule for which the maximum completion time, or makespan, is

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minimized. Using the notation of Hoogeveen et al. (1991), this problem is denoted $F2(P)\,||\,C_{\max}$.

The $F2(P)\,||\,C_{\max}$ might be viewed as a generalization of two fundamental scheduling problems. Indeed, if all processing times at Z_2 are equal to zero then the problem amounts to solving a parallel machine problem with identical processors ($P\,||\,C_{\max}$) which is $\mathcal{NP}$-hard in the ordinary sense (Karp 1972). Also, if both stages contain a single machine, then the problem reduces to the two-machine flow shop problem ($F2\,||\,C_{\max}$) which is solvable in $O(n \log n)$ time. However, Gupta et al. (1997) show that when at least one stage contains multiple machines (i.e. $\max(m_1, m_2) > 1$) then the problem turns out to be $\mathcal{NP}$-hard in the strong sense. Moreover, a further indication of the intrinsic hardness of this problem is that even its preemptive version is $\mathcal{NP}$-hard in the strong sense (Hoogeveen et al. 1996).

The $F2(P)\,||\,C_{\max}$ and its variants are adequate models for several manufacturing settings. Practical applications are discussed in Lin and Liao (2003), Narasimhan and Pawnwalker (1984), and Sherali et al. (1990), to quote just a few. The theoretical and practical importance of the $F2(P)\,||\,C_{\max}$ motivated several researchers to investigate it. In particular, most efforts have been focused on developing and analyzing heuristic algorithms with worst-case error bounds. These methods are based on a predefined ordering of the jobs and have a low complexity of $O(n \log n)$ (see for instance (Buten and Shen 1973; Langston 1987; Lee and Vairaktarakis 1994; Sriskandarajah and Sethi 1989). In particular, in Lee and Vairaktarakis (1994), the authors describe a heuristic with an error bound of $2 - \frac{1}{\max(m_1, m_2)}$. Recently, a remarkable result has been obtained by Schuurman and Woeginger (2000) who demonstrated the existence of a polynomial time approximation scheme for this challenging problem.

On contrast, the literature dealing with the exact solution of $F2(P)\,||\,C_{\max}$ is surprisingly scant. Indeed, the only relevant work that we are aware is the branch-and-bound algorithm described by Gupta et al. (1997) who addressed the particular case where the second stage contains a single machine ($m_2 = 1$). They presented computational tests with up to 250 jobs. In addition, several authors developed exact procedures for the multiple-stage hybrid flow shop problem (see Brah and Hunsucker 1991; Carlier and Néron 2000; Moursli and Pochet 2000; Néron et al. 2001; Perregaard 1995; Portman et al. 1998; Rajendran and Chaudhuri 1992; Salvador 1973). Néron et al. (2001) describe an exact approach which outperforms all previous ones and report the optimal solution of small-sized instances with up to 15 jobs, 5 stages, and 3 machines in each stage. For an overview on exact methods for the multiple-stage hybrid flow shop problem the reader is referred to the survey paper of Kis and Pesch (2004).

In this paper, we present an effective branch-and bound algorithm which has been specifically designed for solving the $F2(P)\,||\,C_{\max}$ problem with an arbitrary number of machines in each stage. However, although our approach could be easily modified to handle the particular case where one of the two stages contains a single machine, we assume, for the sake of simplicity, that each stage contains at least two parallel machines (i.e. $\min(m_1, m_2) \geq 2$). A distinctive feature of our branch-and-bound is that the evaluation of terminal nodes of the search tree requires the optimal solution of a $P\left|r_j\right|C_{\max}$. However, although this problem is known to be intractable, we provide evidence that its hardness doesn't preclude its effectiveness for lower bound computation. Other features that are peculiar to our procedure include