Efficient Collision Detection for Moving Ellipsoids Using Separating Planes

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Abstract

We present a simple, accurate and efficient algorithm for collision detection among moving ellipsoids. Its efficiency is attributed to two results: (i) a simple algebraic test for the separation of two ellipsoids, and (ii) an efficient method for constructing a separating plane between two disjoint ellipsoids. Interframe coherence is exploited by using the separating plane to reduce collision detection to simpler subproblems of testing for collision between the plane and each of the ellipsoids. Compared with previous algorithms (such as the GJK method) which employ polygonal approximation of ellipsoids, our algorithm demonstrates comparable computing speed and much higher accuracy.

AMS Subject Classifications: 65D17, 68U07, 68U05.

Keywords: Collision detection, ellipsoids, algebraic test, separating plane, characteristic equation, self-polar.

1. Introduction

Collision detection has many important applications in computer graphics, including the simulation of virtual environments, computer animation and, in particular, 3D computer games [5], [8], [9], [15], [18]. Ellipsoids are frequently used for exact shape representation (e.g. in molecule simulation) and also as a tight bounding shape for many natural objects and organic forms used in character modeling [2]. Thus efficient algorithms for detecting collision among ellipsoids have considerable potential.

We represent the interiors of two ellipsoids $A$ and $B$ by the inequalities $X^TAX < 0$ and $X^TBX < 0$, where $A$ and $B$ are $4 \times 4$ real symmetric matrices and $X = (x, y, z, w)^T$ represents a point in homogeneous coordinates.

A simple algebraic condition for the separation of two ellipsoids is established by Wang et al. [26]. Given two ellipsoids $A : X^TAX = 0$ and $B : X^TBX = 0$, the quartic equation $f(\lambda) = \det(\lambda A + B) = 0$ is called the characteristic equation of $A$ and $B$. Two ellipsoids are said to be disjoint if they do not have a common boundary or interior point.
Proposition 1 [26]. Let \( A \) and \( B \) be two ellipsoids with the characteristic equation
\[
f(\lambda) = 0.
\]
Then

1. \( A \) and \( B \) are disjoint if and only if \( f(\lambda) = 0 \) has two distinct positive roots;
2. \( A \) and \( B \) touch each other externally if and only if \( f(\lambda) = 0 \) has a positive
double root.

Figure 1a shows two disjoint ellipsoids. Note that their characteristic equation
has two distinct positive roots. In Fig. 1b, two ellipsoids overlap and their
characteristic equation has no positive root.

Combining this result with a simple method for constructing a separating plane
for two disjoint ellipsoids, we devise an efficient algorithm for detecting collisions
between two moving ellipsoids. An arbitrary number of moving ellipsoids can also
be dealt with by repeated pairwise application of the algorithm.

It is well known that the efficiency of collision detection can be greatly improved
by using a separating plane [1]. Once a plane separating two ellipsoids is found,
there can be no collision between the ellipsoids until one of them collides with the
separating plane. Thus the original problem is reduced to two simpler subprob-
lems of searching for an intersection between a plane and an ellipsoid. Applying
an affine transformation, an ellipsoid and a plane can be reduced to a sphere and a
plane, and each of these subproblems then becomes equivalent to computing the
distance between the center of the sphere and a plane.

Since ellipsoids are preserved under affine motion, our approach can be applied to
ellipsoids that are moving and deforming under affine transformation. This is an
important advantage over specialized algorithms that work only for simple geo-
metric shapes such as axis-aligned boxes, spheres, cylinders, cones, or tori; such
algorithms may not be generalized when affine motions are used.

The remainder of this paper is organized as follows. In Section 2, we briefly
review related previous work. In Section 3, we develop a method for constructing
a separating plane for two disjoint ellipsoids. In Section 4, we present our com-
plete collision detection algorithm. In Section 5, experimental results are dis-
cussed. Section 6 concludes this paper.

2. Related Work

In the past, collision detection for ellipsoids was usually performed by faceting,
and then applying a collision detection package appropriate to general convex
polyhedra, such as GJK [7], I-COLLIDE [4], or V-Clip [14]. A drawback with this
approach is that accuracy and efficiency are compromised by polyhedral
approximation.

Rimon and Boyd [17] present an efficient numerical technique for computing a
quasi-distance, which they call the ‘margin’, between two disjoint ellipsoids, using
an incremental approach. Based on line geometry, Sohn et al. [20] devised a