IMPULSIVE SEMILINEAR NEUTRAL FUNCTIONAL DIFFERENTIAL INCLUSIONS WITH MULTIVALUED JUMPS

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Abstract. In this paper we establish sufficient conditions for the existence of mild solutions and extremal mild solutions for some densely defined impulsive semilinear neutral functional differential inclusions in separable Banach spaces. We rely on a fixed point theorem for the sum of completely continuous and contraction operators.

Keywords: impulsive semilinear neutral functional differential equation, densely defined operator, infinite delay, phase space, fixed point, mild solutions, extremal mild solution

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1. Introduction

In this paper we are concerned with the existence of mild solutions and extremal mild solutions defined on a compact real interval for first order impulsive semilinear neutral functional inclusions in a separable Banach space. More precisely, we will consider the following first order impulsive semilinear neutral functional differential inclusions:

$$\frac{\mathrm{d}}{\mathrm{d}t}[y(t)-g(t,y_t)]-A[y(t)-g(t,y_t)]\in F(t,y_t),\quad \text{a.e. } t\in J=[0,b],\ t\neq t_k,\ k=1,\dots,m, \tag{1}$$

$$\Delta y|_{t=t_k}\in I_k(y(t_k^-)),\qquad k=1,\dots,m, \tag{2}$$

$$y(t)=\varphi(t),\quad t\in(-\infty,0], \tag{3}$$

where $F\colon J\times D\to\mathcal{P}(E)$ is a compact and convex valued multivalued map, $g\colon J\times D\to E$ is a given function, $0=t_0<t_1<\ldots<t_m<t_{m+1}=b$, $\varphi\in D$, where

D is the phase space that will be specified later, $I_k\colon E \to \mathcal{P}(E)$, $k = 1, 2, \ldots, m$, are bounded valued multivalued maps, $\mathcal{P}(E)$ is the collection of all subsets of E, $A\colon D(A) \subset E \to E$ is a densely defined closed linear operator on E, and E is a real separable Banach space with a norm $|\cdot|$. For any function y defined on $(-\infty, b] \setminus \{t_1, t_2, \ldots, t_m\}$ and any $t \in J$, we denote by y_t the element of D defined by

$$y_t(\theta) = y(t + \theta), \quad \theta \in (-\infty, 0].$$

Functional and neutral functional differential equations arise in a variety of areas of biological, physical, and engineering applications, see, for example, the books of Hale [21], Hale and Verduyn Lunel [23], Kolmanovskii and Myshkis [32], Kuang [33] and Wu [45], and the references therein. Impulsive differential and partial differential equations are used to describe various models of real processes and phenomena studied in physics, chemical technology, population dynamics, biotechnology, and economics. That is why in recent years they have been the object of investigations. We refer to the monographs of Bainov and Simeonov [7], Benchohra *et al* [10], Lakshmikantham *et al* [34], and Samoilenko and Perestyuk [42], where numerous properties of their solutions are studied, and a detailed bibliography is given. Semilinear functional differential equations and inclusions with or without impulses have been extensively studied where the operator A generates a C_0-semigroup. The existence and uniqueness, among other things, have been derived; see the books of Ahmed [3], [4], Benchohra *et al* [9], Heikkila and Lakshmikantam [24], Kamenskii *et al* [29] and the papers by Ahmed [5], [6], Liu [37], and Rogovchenko [40], [41]. In [2] Abada *et al* have studied the controllability of a class of impulsive semilinear functional differential inclusions in Fréchet spaces by means of the extrapolation method ([13], [18]), and in [1] the existence of mild and extremal mild solutions for first-order semilinear densely defined impulsive functional differential inclusions in separable Banach spaces with local and nonlocal conditions has been considered. To the best of our knowledge, there are very few results for impulsive evolution inclusions with multivalued jump operators; see [1], [11], [38]. The notion of the phase space D plays an important role in the study of both the qualitative and quantitative theory. A usual choice is a semi-normed space satisfying suitable axioms, which was introduced by Hale and Kato [22] (see also Kappel and Schappacher [30] and Schumacher [43]). For a detailed discussion on this topic we refer the reader to the book by Hino *et al* [27]. For the case where the impulses are absent (i.e. $I_k = 0$, $k = 1, \ldots, m$), an extensive theory has been developed for the problem (1)–(3). We refer to Belmekki *et al* [8], Corduneanu and Lakshmikantham [12], Hale and Kato [22], Hino *et al* [27], Lakshmikantham *et al* [35] and Shin [44]. The study of first order abstract neutral functional differential equations with unbounded delay was initiated by Hernandez