

# The Mathematical Universe

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**Abstract** I explore physics implications of the *External Reality Hypothesis* (ERH) that there exists an external physical reality completely independent of us humans. I argue that with a sufficiently broad definition of mathematics, it implies the *Mathematical Universe Hypothesis* (MUH) that our physical world is an abstract mathematical structure. I discuss various implications of the ERH and MUH, ranging from standard physics topics like symmetries, irreducible representations, units, free parameters, randomness and initial conditions to broader issues like consciousness, parallel universes and Gödel incompleteness. I hypothesize that only computable and decidable (in Gödel’s sense) structures exist, which alleviates the cosmological measure problem and may help explain why our physical laws appear so simple. I also comment on the intimate relation between mathematical structures, computations, simulations and physical systems.

## 1 Introduction

The idea that our universe is in some sense mathematical goes back at least to the Pythagoreans, and has been extensively discussed in the literature (see, *e.g.*, [1–24]). Galileo Galilei stated that the Universe is a grand book written in the language of mathematics, and Wigner reflected on the “unreasonable effectiveness of mathematics in the natural sciences” [2]. In this essay, I will push this idea to its extreme and argue that our universe *is* mathematics in a well-defined sense. After elaborating on this hypothesis and underlying assumptions in Sect. 2, I discuss a variety of its implications in Sects. 3–7. This paper can be thought of as the sequel to one I wrote in 1996 [11], clarifying and extending the ideas described therein.

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## 2 The Mathematical Universe Hypothesis

### 2.1 The External Reality Hypothesis

In this section, we will discuss the following two hypotheses and argue that, with a sufficiently broad definition of mathematical structure, the former implies the latter.

**External Reality Hypothesis (ERH):** *There exists an external physical reality completely independent of us humans.*

**Mathematical Universe Hypothesis (MUH):** *Our external physical reality is a mathematical structure.*

Although many physicists subscribe to the ERH and dedicate their careers to the search for a deeper understanding of this assumed external reality, the ERH is not universally accepted, and is rejected by, *e.g.*, metaphysical solipsists. Indeed, adherents of the Copenhagen interpretation of quantum mechanics may reject the ERH on the grounds that there is no reality without observation. In this paper, we will assume that the ERH is correct and explore its implications. We will see that, although it sounds innocuous, the ERH has sweeping implications for physics if taken seriously.

Physics theories aim to describe how this assumed external reality works. Our most successful physics theories to date are generally regarded as descriptions of merely limited aspects of the external reality. In contrast, the holy grail of theoretical physics is to find a *complete* description of it, jocularly referred to as a “Theory of Everything”, or “TOE”.

The ERH implies that for a description to be complete, it must be well-defined also according to non-human sentient entities (say aliens or future supercomputers) that lack the common understanding of concepts that we humans have evolved, *e.g.*, “particle”, “observation” or indeed any other English words. Put differently, such a description must be expressible in a form that is devoid of human “baggage”.

### 2.2 Reducing the Baggage Allowance

To give a few examples, Fig. 1 illustrates how various theories can be crudely organized in a family tree where each might, at least in principle, be derivable from more fundamental ones above it. All these theories have two components: mathematical equations and “baggage”, words that explain how they are connected to what we humans observe and intuitively understand. Quantum mechanics as usually presented in textbooks has both components: some equations as well as three fundamental postulates written out in plain English. At each level in the hierarchy of theories, new concepts (*e.g.*, protons, atoms, cells, organisms, cultures) are introduced because they are convenient, capturing the essence of what is going on without recourse to the more fundamental theory above it. It is important to remember, however, that it is we humans who introduce these concepts and the words for them: in principle, everything could have been derived from the fundamental theory at the top of the tree, although such an extreme reductionist approach appears useless in practice. Crudely speaking,