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Enhancements to Two Exact Algorithms for Solving the Vertex P-Center Problem

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Abstract. Enhancements to two exact algorithms from the literature to solve the vertex P-center problem are proposed. In the first approach modifications of some steps are introduced to reduce the number of ILP iterations needed to find the optimal solution. In the second approach a simple enhancement which uses tighter initial lower and upper bounds, and a more appropriate binary search method are proposed to reduce the number of subproblems to be solved. These ideas are tested on two well known sets of problems from the literature (i.e., OR-Lib and TSP-Lib problems) with encouraging results.

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1. Introduction

The P-center problem (also known as the minimax problem) is to locate P facilities and assign clients to them so as to minimise the maximum distance between a client and the facility to which it is assigned. This well known location problem, which was first introduced by Hakimi [7, 8], has several applications including the location of emergency facilities such as ambulance stations and firehouses. There are several possible variations of the basic model. If facility locations are restricted to the nodes of the network, the problem is referred to as a *vertex center problem.* Center problems which allow facilities to be located anywhere on the network are known as *absolute center problems.* Both versions can be either *weighted* or *unweighted.* In the weighted problem, the distances between demand nodes and facilities are multiplied by a weight usually associated with the demand node. As an example, the weight might represent the population or the importance of a node. In the unweighted problem, all demand nodes are treated equally.

To formulate the vertex P-center problem, we define:

$I =$ set of demand nodes, $I = \{1, \dots, N\}$,
$J =$ set of candidate facility sites, $J = \{1, \dots M\}$,
$d_{ij} =$ distance between demand node $i \in I$ and candidate site $j \in J$,
$P =$ number of facilities to be located,

$$w_j = \begin{cases} 1 & \text{if a facility is located at candidate site } j \in J, \\ 0 & \text{otherwise,} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if demand node } i \in I \text{ is assigned to an open facility at candidate} \\ & \text{site } j \in J, \\ 0 & \text{otherwise,} \end{cases}$$

$D =$ maximum distance (or time) between a demand node and the nearest facility (D is also referred to as the covering distance or time).

The binary linear programming formulation of the vertex P-center problem is as follows:

$$\text{Minimise} \quad D \tag{1}$$

subject to:

$$\sum_{j \in J} Y_{ij} = 1 \quad \forall i \in I, \tag{2}$$

$$Y_{ij} \leqslant w_j \quad \forall i \in I, j \in J, \tag{3}$$

$$\sum_{j \in J} w_j = P, \tag{4}$$

$$D \geqslant \sum_{j \in J} d_{ij} Y_{ij} \quad \forall i \in I, \tag{5}$$

$$w_j, Y_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J. \tag{6}$$

The objective function (1) minimises the maximum distance between each demand node and its closest open facility. Constraint (2) ensures that each demand node is assigned to exactly one facility, while constraints (3) restrict demand nodes to be assigned to open facilities. Constraint (4) stipulates that P facilities are to be located. Constraints (5) define the maximum distance between any demand node i and the nearest facility at node j. Finally, constraints (6) refer to integrality constraints.

For fixed values of P, the vertex P-center problem can be solved in polynomial time. This can be done by evaluating each of the $O(N^p)$ possible combinations of P facility sites. Evaluating each of these can be done in polynomial time [3] though it may take a considerable amount of CPU time. For variable values of P, the P-center problem is NP-hard [10].

Different authors have used an auxiliary problem (e.g., the Set Covering Problem; SCP) to solve the P-center problem optimally. The objective of the SCP is to find the minimum number of facilities and their locations so that each demand point has to be served by a facility within a specified maximum response time (or distance) which can be referred to as radius. The solution to this problem can be easily found by solving its linear programming relaxation with occasional branch and bound applications. However, for large problems the size of the relaxed version