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Energetic reasoning revisited: application to parallel machine scheduling

Lotfi Hidri · Anis Gharbi · Mohamed Haouari

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Abstract We consider the problem of minimizing makespan on identical parallel machines subject to release dates and delivery times. We present several new feasibility tests and adjustment techniques that consistently improve the classical energetic reasoning approach. Computational results carried out on a set of hard instances provide strong evidence that the performance of a state-of-the-art exact branch-and-bound algorithm is substantially improved through embedding the proposed enhanced energetic reasoning.

Keywords Scheduling · Release dates · Due dates · Makespan · Feasibility and adjustment procedures · Energetic reasoning · Branch-and-bound

1 Introduction

We investigate the problem of scheduling identical parallel machines with release dates and delivery times. This problem is denoted by $P|r_j, q_j|C_{\max}$ and is formally described as follows: A set $\mathcal{J}$ of n jobs has to be scheduled on m identical parallel machines (with $n > m \geq 2$). Each job j ($1 \leq j \leq n$) has a processing time p_j, a release date (or head) r_j on which the job becomes available for processing, and a delivery time (or tail) q_j that must elapse between its completion on the machine and its exit from the system. Each machine processes at most one job at one time and each job cannot be processed by more than one machine at one time. We assume that preemption is not allowed and that all machines are available from time zero onwards. The objective is to find a schedule that minimizes the makespan.

This strongly $\mathcal{NP}$-hard problem has been intensely investigated in the machine scheduling literature. Carlier (1987) proposed a first exact branch-and-bound algorithm. Gharbi and Haouari (2002) improved this algorithm by embedding new lower and upper bounds as well as an efficient preprocessing algorithm. Gharbi and Haouari (2005) have proposed a new exact branch-and-bound algorithm that is based on the implementation of a strong max-flow based lower bound as well as effective dominance rules. During the last decade, several investigations have focused on deriving lower bounding strategies for the $P|r_j, q_j|C_{\max}$. Carlier and Pinson (1998) proposed the so-called Jackson's pseudo-preemptive schedule and showed that it yields a valid lower bound for the $P|r_j, q_j|C_{\max}$. Haouari and Gharbi (2003) improved the preemptive lower bound of Horn (1974) by introducing the semi-preemptive lower bound. Lately, Tercinet et al. (2004) proposed a stronger lower bound that is both based on preemptive scheduling and energetic reasoning. Haouari and Gharbi (2004) proposed a general procedure which aims at improving the quality of a given lower bound by considering subsets of jobs and machines. Recently, Vandevelde et al. (2005) introduced the so-called adjusted lower bound which takes into account in a clever way the non-preemption constraint. Moreover, research efforts

L. Hidri
Combinatorial Optimization Research Group—ROI,
Ecole Polytechnique de Tunisie, BP 743, 2078 La Marsa, Tunisia
e-mail: lotfi162002@yahoo.fr

A. Gharbi (✉)
Department of Industrial Engineering, College of Engineering,
King Saud University, PO Box 800, Riyadh 11421, Saudi Arabia
e-mail: a.gharbi@ksu.edu.sa

M. Haouari
Faculty of Economics and Administrative Sciences,
Özyegin University, Istanbul, Turkey
e-mail: mohamed.haouari@ept.rnu.tn

have been also devoted to the development of heuristic procedures. Gusfield (1984) and Carlier (1987) investigated the worst-case performance of Jackson's algorithm. This latter is a constructive heuristic which consists in scheduling the available job with the largest tail on the first available machine. Finally, Gharbi and Haouari (2007) developed an effective optimization-based heuristic which consistently provides very near-optimal solutions for instances with up to 2000 jobs and 100 machines.

In this paper, we consider the decision variant of the $P|r_j,q_j|C_{\max}$ which consists in checking the existence of a feasible schedule with makespan equal to a fixed value C. This decision problem is denoted by $P|r_j,d_j|-$, where $d_j = C - q_j$ for all $j \in \mathcal{J}$, and it amounts to checking the existence of a feasible schedule such that each job $j \in \mathcal{J}$ is processed nonpreemptively within its prescribed time window $[r_j, d_j]$, respectively. The main contribution of this work is the development of new feasibility tests and adjustment procedures (or, consistency tests) for this problem. Adjustment techniques aim at tightening the time windows that are associated with each job, respectively. The objective is threefold: detecting infeasibility, improving the values of the lower bounds, and more importantly to enhance the efficiency of exact algorithms. Since the pioneering work of Carlier and Pinson (1989) on the job-shop problem, the use of adjustment techniques has been extensively investigated by many authors (Brucker et al. 1994; Carlier and Pinson 1990, 1994). In addition, Baptiste et al. (1999) and Néron et al. (2001) proposed adjustment techniques for the cumulative problem and the hybrid flow shop problem, respectively. In this paper, we propose several improvements of the so-called *energetic reasoning* which has been initially proposed for cumulative scheduling by Erschler et al. (1991). Our computational experiments show that, in contrast to the *classical* energetic reasoning, the proposed *enhanced* energetic reasoning substantially improves the performance of the best existing branch-and-bound algorithm for the $P|r_j,q_j|C_{\max}$. Also, it is worth emphasizing that although this paper mainly focuses on parallel machine scheduling, the proposed techniques can be extended to more complex scheduling problems including cumulative scheduling and job shop scheduling problems.

The paper is organized as follows. In Sect. 2, the energetic reasoning is briefly described. Section 3 presents the main contribution: enhanced energetic reasoning-based feasibility and adjustment procedures are detailed. In Sect. 4, the results of a computational study are reported. Finally, some concluding remarks are provided.

2 The energetic reasoning

Initiated by the paper of Lahrichi (1982), *Energetic Reasoning* (ER) has been originally developed by Erschler et al. (1991) to solve cumulative scheduling problems. The energetic approach which has been formalized and assessed both from a theoretical and an experimental point of view by Baptiste et al. (1999), aims at developing feasibility (or as more commonly referred to, satisfiability) tests and time-bound adjustments to ensure that either a given schedule is infeasible or to derive some necessary conditions that any feasible schedule must satisfy. Moreover, Dorndorf et al. (2000) investigate the use of energetic reasoning within the broader issue of consistency tests (i.e., domain reduction techniques). Since its inception, energetic reasoning has gained popularity and has been used for solving more complex scheduling problems including the hybrid flow shop problem (Néron et al. 2001) and the resource constrained project scheduling problem (Baptiste and Demassey 2004) as well.

For the sake of completeness, we briefly recall the essentials of energetic reasoning. Given a time interval $[t_1, t_2]$, the energetic reasoning is based on the computation of the mandatory part of each job, denoted by its *work*, that must be processed in any feasible schedule within $[t_1, t_2]$. This work, denoted by $W_j(t_1, t_2)$, is equal to the minimum between the *left-work* (denoted by $W_j^l(t_1, t_2)$) and the *right-work* (denoted by $W_j^r(t_1, t_2)$) of the job, which are computed by either left-shifting or right-shifting the job on its time window $[r_j, d_j]$; i.e., a job either starts at r_j or finishes at d_j. More precisely, the left-work (resp., the right-work) of a job j over $[t_1, t_2]$, is defined as the part of the job that must be processed between t_1 and t_2 if the job starts at its release date (resp., finishes at its deadline). Néron et al. (2001) proposed the following formulae for the computation of the left, right and total work of a job j over $[t_1, t_2]$:

$$W_j^l(t_1, t_2) = \min\bigl(t_2 - t_1, p_j, \max(0, r_j + p_j - t_1)\bigr), \quad (1)$$

$$W_j^r(t_1, t_2) = \min\bigl(t_2 - t_1, p_j, \max(0, t_2 - d_j + p_j)\bigr), \quad (2)$$

$$W_j(t_1, t_2) = \min\bigl(W_j^l(t_1, t_2), W_j^r(t_1, t_2)\bigr). \quad (3)$$

The total work over the time interval $[t_1, t_2]$ is defined by $W(t_1, t_2) = \sum_{j \in \mathcal{J}} W_j(t_1, t_2)$. Clearly, the instance is infeasible if $W(t_1, t_2) > m(t_2 - t_1)$.

Moreover, the time bounds of a job j may be adjusted as follows. Let $s_j(t_1, t_2) = m(t_2 - t_1) - W(t_1, t_2) + W_j(t_1, t_2)$ denote the slack of a job j over $[t_1, t_2]$, i.e., the maximum amount of time that might be allocated to job j during the time interval $[t_1, t_2]$. Let $\bar{r}_j$ and $\bar{d}_j$ denote the adjusted values of r_j and d_j, respectively. Then,

- If $s_j(t_1, t_2) < W_j^l(t_1, t_2)$ then $\bar{r}_j = \max(r_j, t_2 - s_j(t_1, t_2))$.
- If $s_j(t_1, t_2) < W_j^r(t_1, t_2)$ then $\bar{d}_j = \min(d_j, t_1 + s_j(t_1, t_2))$.

Baptiste et al. (1999) prove that the only relevant values of t_1 and t_2 (with $t_1 < t_2$) that need to be considered are the following ones: