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Reasoning defeasibly about probabilities

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Abstract In concrete applications of probability, statistical investigation gives us knowledge of some probabilities, but we generally want to know many others that are not directly revealed by our data. For instance, we may know $\text{prob}(P/Q)$ (the probability of P given Q) and $\text{prob}(P/R)$, but what we really want is $\text{prob}(P/Q\&R)$, and we may not have the data required to assess that directly. The probability calculus is of no help here. Given $\text{prob}(P/Q)$ and $\text{prob}(P/R)$, it is consistent with the probability calculus for $\text{prob}(P/Q\&R)$ to have any value between 0 and 1. Is there any way to make a reasonable estimate of the value of $\text{prob}(P/Q\&R)$? A related problem occurs when probability practitioners adopt undefended assumptions of statistical independence simply on the basis of not seeing any connection between two propositions. This is common practice, but its justification has eluded probability theorists, and researchers are typically apologetic about making such assumptions. Is there any way to defend the practice? This paper shows that on a certain conception of probability—nomic probability—there are principles of "probable probabilities" that license inferences of the above sort. These are principles telling us that although certain inferences from probabilities to probabilities are not deductively valid, nevertheless the second-order probability of their yielding correct results is 1. This makes it defeasibly reasonable to make the inferences. Thus I argue that it is defeasibly reasonable to assume statistical independence when we have no information to the contrary. And I show that there is a function $\Upsilon(r, s, a)$ such that if $\text{prob}(P/Q) = r$, $\text{prob}(P/R) = s$, and $\text{prob}(P/U) = a$ (where U is our background knowledge) then it is defeasibly reasonable to expect that $\text{prob}(P/Q\&R) = \Upsilon(r, s, a)$. Numerous other defeasible inferences are licensed by

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similar principles of probable probabilities. This has the potential to greatly enhance the usefulness of probabilities in practical application.

Keywords Probability · Statistical independence · Defeasible reasoning · Direct inference · Nomic probability · Epistemology

1 The problem of sparse probability knowledge

The use of probabilities is ubiquitous in philosophy, science, engineering, artificial intelligence, economics, and many other disciplines. It is generally supposed that the logical and mathematical structure of probabilities is well understood, and completely characterized by the probability calculus. The probability calculus is typically identified with some form of Kolmogoroff's axioms, often supplemented with an axiom of countable additivity. Mathematical probability theory is a mature subdiscipline of mathematics based upon these axioms, and forms the mathematical basis for most applications of probabilities in the sciences.

There is, however, a problem with the supposition that this is all there is to the logical and mathematical structure of probabilities. The uninitiated often suppose that if we know a few basic probabilities, we can compute the values of many others just by applying the probability calculus. Thus it might be supposed that familiar sorts of statistical inference provide us with our basic knowledge of probabilities, and then appeal to the probability calculus enables us to compute other previously unknown probabilities. The picture is of a kind of foundations theory of the epistemology of probability, with the probability calculus providing the inference engine that enables us to get beyond whatever probabilities are discovered by direct statistical investigation.

Regrettably, this simple image of the epistemology of probability cannot be correct. The difficulty is that the probability calculus is not nearly so powerful as the uninitiated suppose. If we know the probabilities of some basic propositions P, Q, R, S,…, it is rare that we will be able to compute, just by appeal to the probability calculus, a unique value for the probability of some logical compound like $((P\&Q) \vee (R\&S))$. To illustrate, suppose we know that $\text{PROB}(P) = .7$ and $\text{PROB}(Q) = .6$. What can we conclude about $\text{PROB}(P\&Q)$? All the probability calculus enables us to infer is that $.3 \leq \text{PROB}(P\&Q) \leq .6$. That does not tell us much. Similarly, all we can conclude about $\text{PROB}(P \vee Q)$ is that $.7 \leq \text{PROB}(P \vee Q) \leq 1.0$. In general, the probability calculus imposes constraints on the probabilities of logical compounds, but it falls far short of enabling us to compute unique values.

Unless we come to a problem already knowing a great deal about the relevant probabilities, the probability calculus will not enable us to compute the values of unknown probabilities that subsequently become of interest to us. Suppose a problem is described by logical compounds of a set of simple propositions $P_1, \ldots, P_n$. Then to be able to compute the probabilities of all logical compounds of these simple propositions, what we must generally know is the probabilities of every conjunction of the form $\text{PROB}((\sim)P_1\&\ldots\&(\sim)P_n)$. The tildes enclosed in parentheses can be either present or absent. These n-fold conjunctions are called *Boolean conjunctions*, and jointly they constitute a "partition". Given fewer than all but one of them, the only constraint the probability calculus imposes on the probabilities of the remaining