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IMPULSIVE DIFFERENTIAL INCLUSIONS INVOLVING EVOLUTION OPERATORS IN SEPARABLE BANACH SPACES

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We present some results on the existence of mild solutions and study the topological structures of the sets of solutions for the following first-order impulsive semilinear differential inclusions with initial and boundary conditions:

$$y'(t) - A(t)y(t) \in F(t, y(t)) \quad \text{for a.e. } t \in J \setminus \{t_1, \ldots, t_m, \ldots\},$$

$$y(t_k^+) - y(t_k^-) = I_k(y(t_k^-)), \quad k = 1, \ldots,$$

$$y(0) = a$$

and

$$y'(t) - A(t)y(t) \in F(t, y(t)) \quad \text{for a.e. } t \in J \setminus \{t_1, \ldots, t_m, \ldots\},$$

$$y(t_k^+) - y(t_k^-) = I_k(y(t_k^-)), \quad k = 1, \ldots,$$

$$Ly = a,$$

where $J = \mathbb{R}_+$, $0 = t_0 < t_1 < \ldots < t_m < \ldots$, $m \in \mathbb{N}$, $\lim_{k\to\infty} t_k = \infty$, $A(t)$ is the infinitesimal generator of a family of evolution operators $U(t,s)$ in a separable Banach space E and F is a set-valued mapping. The functions I_k characterize the jumps of solutions at the impulse points t_k, $k = 1, \ldots$. The mapping $L: PC_b \to E$ is a bounded linear operator. We also investigate the compactness of the set of solutions, some regularity properties of the operator solutions, and the absolute retract.

1. Introduction

Differential equations with impulses were considered for the first time by Milman and Myshkis [47]. This paper was followed by a period of active research culminated with the monograph by Halanay and Wexler [34]. Numerous phenomena and evolution processes in the field of physics, chemical technology, population dynamics, and natural sciences may change their state abruptly or be subject to short-term perturbations (see, e.g., [2, 42, 43] and the references therein). These perturbations may be regarded as impulses. Impulsive problems also arise in various applications in communications, chemical technology, mechanics (jump discontinuities of the velocity), electrical engineering, medicine, and biology. These perturbations can also be regarded as impulses. Thus, in the periodic treatment of some diseases, impulses correspond to the administration of drug treatment. In the

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environmental sciences, impulses correspond to seasonal changes in the water level of artificial basins. Their models are described by impulsive differential equations and inclusions. Various mathematical results (existence, asymptotic behavior, ...) have been obtained up to now (see [4, 10, 12, 44, 51, 54, 55] and the references therein).

Given a real separable Banach space E with norm $\|\cdot\|$, we consider the following problem:

$$y'(t) - A(t)y(t) \in F(t, y(t)), \quad \text{for a.e. } t \in J \setminus \{t_1, \ldots, t_m, \ldots\},$$

$$\Delta y_{t=t_k} = I_k(y(t_k^-)), \quad k = 1, \ldots, \tag{1}$$

$$y(0) = a \in E,$$

where $J = \mathbb{R}_+$, $0 = t_0 < t_1 < \ldots < t_m < t_{m+1} \ldots$ $(m \in \mathbb{N})$, $\lim_{k\to\infty} t_k = \infty$. $F\colon J \times E \to \mathcal{P}(E)$ is a multivalued map, and $A(t)$ is the infinitesimal generator of a family of evolution operators $\{U(t,s)\}$. We always assume that the operator $A(t)$ is closed and densely defined in its domain $D(A(t))$, which is independent of t, $I_k \in C(E, E)$, $k = 1, \ldots, m$, $\Delta y|_{t=t_k} = y(t_k^+) - y(t_k^-)$, and $y(t_k^+) = \lim_{h\to 0^+} y(t_k + h)$ and $y(t_k^-) = \lim_{h\to 0^+} y(t_k - h)$ stand for the right and left limits of $y(t)$ at $t = t_k$, respectively.

Later, we study the following impulsive boundary-value problems:

$$y'(t) - A(t)y(t) \in F(t, y(t)), \quad \text{for a.e. } t \in J \setminus \{t_1, \ldots, t_m, \ldots\},$$

$$y(t_k^+) - y(t_k^-) = I_k(y(t_k^-)), \quad k = 1, \ldots, \tag{2}$$

$$Ly = a,$$

where $L\colon PC_b \to E$ is a bounded linear operator.

Numerous properties connected with the solution of differential equations and inclusions, such as the stability or oscillations, require global properties of the solutions. This is the main motivation for seeking sufficient conditions for the global existence of solutions of the impulsive differential equations and inclusions. In this direction, some problems were discussed by Baghli and Benchohra [7–9], Graef and Ouahab [26, 28], Guo [30, 31], Guo and Liu [32], Henderson and Ouahab [35–37], Marino et al. [46], Ouahab [49, 50], Stamov and Stamova [56], Weng [60], and Yan [61, 62].

In case where E is a finite-dimensional space and J is a compact interval, some results concerning the existence of solutions for problems (1) and (2) in a particular case $Ay = \lambda y$, $\lambda \in \mathbb{R}$, were obtained in [11, 13, 27]. Very recently, some existence results and the sets of solutions on unbounded intervals were studied by Djebali et al. [1, 21, 22] for the infinite-dimensional space and the case where A is the infinitesimal generator of a C_0-semigroup and $Ly = a - y(0) + y(b)$. The problems of existence and the sets of solutions for the problems on bounded intervals mentioned above were solved by Djebali et al. [19, 20, 23].

The aim of the present work is to complement and extend some recent results to the case of infinite-dimensional spaces. Moreover, the right-hand side nonlinearity may be either convex or nonconvex. Our approach is based on a nonlinear alternative for compact u.s.c. maps. Then we present some existence results and study the compactness of the set of solutions. Some regularity properties of the operator solutions and the absolute retract (AR) of the solution are also proved.