A stage-structured predator-prey model with Beddington-DeAngelis functional response

Liming Cai · Jingyuan Yu · Guangtian Zhu

Received: 26 September 2006 / Revised: 5 January 2007 / Published online: 5 January 2008
© KSCAM and Springer-Verlag 2008

Abstract In this paper, a stage-structured predator-prey model with Beddington-DeAngelis functional response is proposed and analyzed. It is assumed in the model that the individuals in each specie may belong to one of two classes: the immature and the mature, the age of maturity is represented by a time delay. By using the persistence theory for infinite dimensional systems, necessary and sufficient conditions for the permanence of the system are obtained. By constructing suitable Lyapunov functions and using an iterative technique, a set of easily verifiable sufficient conditions is also obtained for the local asymptotic stability and the global attractiveness of the positive equilibrium of the model.

Keywords Predator-prey system · Stage structure · Beddington-DeAngelis · Permanence · Global attractiveness

Mathematics Subject Classification (2000) 34C25 · 92D25 · 34D20

This work is supported by the National Natural Science Foundation of China (No. 10671166).

L. Cai (✉) 
College of Mathematics and Information Science, Xinyang Normal University, Xinyang 464000, Henan, China 
e-mail: lmcai06@yahoo.com.cn

J. Yu 
Beijing Institute of Information Control, Beijing 100037, China 
e-mail: yujingyuan@biic.net

G. Zhu 
Academy of Mathematics and System Science, CAS, Beijing 100080, China 
e-mail: gtzhu@amss.ac.cn
1 Introduction

In recent years, stage-structured models have received much attention. In [2], Aiello and Freedman proposed and investigated a stage-structured model of single species growth, where the transformation rate of mature is proportional to the existing immature species. The pioneering work of Aiello and Freedman [2] represents a mathematically more careful and ecologically meaningful formulation approach. Many authors have joined the studies on different kinds of stage-structured models and have made much more significant progress (see [3, 7, 13, 14, 16, 18–21]). In particular, in [20], a stage-structured ratio-dependent predator-prey model was proposed and investigated by Wang et al. to show the effect of the duration time of immature predator on the global dynamics of predator-prey system. But their model [20] ignores the duration time of immature prey population.

On the other hand, the dynamic interactions between the predator and the prey have long been one of the dominant themes in mathematical biology due to their universal existence and importance. In the description of the dynamic interactions, a crucial element of all models is the classic definition of predator’s functional responses. There have been several famous functional response types: Holling types I–III [10], Hassel-Varley type [9], Beddington-DeAngelis types [5] and the recent well-known ratio-dependence type [4], etc. As Abrams and Ginzburg [1] even pointed out that precise prey dependence and ratio-dependence will both be rare, while predator dependence will be common. Recent studies show in some cases the Beddington-DeAngelis functional response can provide better descriptions of predator feeding over a range of predator-prey abundances. Thus, some ecological models with Beddington-DeAngelis functional response have recently been discussed [6, 11, 15, 17]. However, the stage-structured predator-prey models of two or more species with such functional response received little attention. Enlightened by the recent work of Aiello and Freedman [2] and Wang et al. [20], in this paper, we consider a stage-structured predator-prey system with Beddington-DeAngelis functional response to study how the stage structure and the predator interference parameter $n$ (or $c_2$) affect the dynamical behaviors of the system.

The organization of the paper is as follows: In the next section, we present our model and discuss the existence of the nonnegative equilibria. In Sect. 2, by using the persistence theory [8], we establish necessary and sufficient conditions for permanence of the system. By applying the characteristic equation theory and constructing Lyapunov functionals, the local stability of the nonnegative equilibria of the system is discussed, respectively, in Sect. 3. In Sect. 4, by using an iterative technique, we discuss the global attractiveness of the positive equilibrium of the system. The paper ends with a discussion and further work.

2 The model and equilibria

In this section, we first present a stage-structured predator-prey model with Beddington-DeAngelis functional response as follows: