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Asymptotic Results for an L^1-norm Kernel Estimator of the Conditional Quantile for Functional Dependent Data with Application to Climatology

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Abstract

In this paper, we study an L^1-norm kernel estimator of the conditional quantile (CQ) of a scalar response variable Y given a random variable (rv) X taking values in a semi-metric space. The almost complete (*a.co.*) consistency and the asymptotic normality of this estimate are obtained when the sample is an α-mixing sequence. We illustrate our methodology by applying the estimator to climatological data.

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1 Introduction

Estimating the CQ constitutes an important statistical topic. It is used to build predictive intervals, as a prediction method by the conditional median and to determine reference curves, predictive intervals etc. It has been widely studied, when the explanatory variable lies within a finite-dimension space (see, e.g., Gannoun et al., 2003 and the references therein).

The goal of this paper is to study a nonparametric estimator of the CQ when the explanatory variable is functional. This is motivated by the increasing number of situations in which the collected data are curves (consecutive discrete recordings are aggregated and viewed as sampled values of

a random curve) where it used to be numbers and vectors. Functional data analysis (see Ferraty and Vieu, 2006) can help to analyze such data sets in a nonparametric framework.

The a.co.[1] consistency and asymptotic normality of the kernel estimator of the CQ were obtained in Ferraty et al. (2005) and Ezzahrioui and Ould Saïd (2008), for the dependent case.

In an earlier contribution (see Laksaci et al., 2009) we established the asymptotic normality of the L^1-norm kernel estimator for the independent case. The present work gives a generalization to the dependent case. The interest comes mainly from the fact that application fields for functional methods need to analyze continuous-time stochastic processes. These statistical motivations are illustrated using a real-data set.

In what follows, we present our estimation procedure and recall the definition of the strong mixing property in Section 2 before giving the hypotheses and stating the main results in Section 3. Section 4 is devoted to an application to a time series prediction problem. An example of a real-data application is considered in Section 5 whereas the proofs of the results are given in the last section.

2 Model

Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be n copies of a random vector, identically distributed as (X, Y) and valued in $\mathcal{F} \times \mathbb{R}$, where $(\mathcal{F}, d)$ is a semi-metric space. In the following, we fix a point $x \in \mathcal{F}$ and a neighborhood N_x of x. We assume that the regular version $F^{x'}$ of the conditional distribution function of Y given $X = x'$ exists for any $x' \in N_x$. Moreover we suppose that F^x has a continuous density f^x with respect to (w.r.t.) Lebesgue's measure over $\mathbb{R}$. Then, for a fixed $p \in (0,1)$, let $t_p(x)$ denote the CQ of order p of F^x which is uniquely defined as the smallest minimum, w.r.t. t, of

$$\Psi_p(x,t) := \mathbb{E}\left[L_p(Y-t)\,|X=x\right]$$

with $L_p(t) = (2p-1)t + |t|$. The kernel estimate of $\Psi_p(x,t)$ is given by

$$\widehat{\Psi}_p(x,t) = \sum_{i=1}^{n} W_{ni}(x) L_p(Y_i - t), \text{ for all } t \in \mathbb{R}$$

[1] We say that a sequence Z_n converges a.co. to Z if and only if, for any $\epsilon > 0$, we have $\sum_n \mathbb{P}\left(|Z_n - Z| > \epsilon\right) < \infty$.