

# Link Disjoint Paths Using Auxiliary Graph Transformation in WDM Networks

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**Abstract** This paper presents a new method for finding two link-disjoint paths in WDM networks under wavelength continuity and lowest cost constraints. Such a problem is considered to be an NP-complete problem, which is only solvable using Integer Linear Programming (ILP). The presented method is based on transforming the original network into an auxiliary network with  $n \times n$  nodes and  $2mn$  links, where  $n$  is the number of nodes and  $m$  is the number of links in the original network, and then applying a modified version of Dijkstra's shortest path algorithm on that network. Despite the larger network size, the execution time of the algorithm is in polynomial order. Considering that the problem is NP-complete, the presented algorithm takes much less time than using ILP, which generally requires exponential time. Yet, it is able to find all available disjoint paths obtainable by ILP.

**Keywords** Disjoint paths · WDM networks · Network survivability · Optimization algorithms

## الخلاصة

تقدم هذه الورقة طريقة جديدة لإيجاد مسارين منفصلي الروابط في شبكات دمج تقسيم الطول الموجي WDM المقيدة بعدم تحويل الطول الموجي بأقل تكلفة ممكنة. وتعد مثل هذه المشكلة غير حدودية كاملة NP complete ويمكن حلها فقط باستخدام البرمجة الخطية الصحيحة. والطريقة المقدمه مبنية على تحويل الشبكة الأصلية إلى شبكة مساعدة بعدد  $n \times n$  من النقاط وعدد  $2mn$  من الروابط، حيث  $n$  هو عدد النقاط، و  $m$  هو عدد الروابط في الشبكة الأصلية، ومن ثم تطبيق نسخة معدلة من خوارزمية ديكسترا لإيجاد المسار الأقصر على تلك الشبكة المساعدة. وعلى الرغم من الزيادة في حجم الشبكة المساعدة فإن وقت تنفيذ الخوارزمية المقدمه في نطاق الحسابات الحدودية. وإذا أخذنا بعين الاعتبار أن المشكلة غير حدودية كاملة، فإن الخوارزمية المقدمه تتطلب وقتاً أقل بكثير من طريقة البرمجة الخطية الصحيحة التي تتطلب عادة وقتاً أسياً، ومع ذلك يمكنها إيجاد جميع المسارات المنفصلة التي يمكن إيجادها باستخدام البرمجة الخطية الصحيحة.

## 1 Introduction

Survivability is an integral part of optical WDM networks design and planning. This is due to the massive transmission capacities of these networks. A single optical fiber link can carry thousands of phone calls per second across an entire city, a desert or a sea. Fiber cables are vulnerable to failures from natural disasters, cable cuts by undersea creatures and other causes. Consequences of high-capacity link failures are severe. Entire regions of the world can go dark due to a couple of fiber cuts [1].

Survivability falls under two main categories: protection (proactive) and restoration (reactive). Protection schemes provide alternative paths between endpoints prior to connection establishment. Restoration schemes attempt to find alternative path after the failure occurs in the primary path. Protection schemes generally provide speedier recovery and thus are preferred over restoration schemes [2]. Protection involves finding a backup path together with the primary path when a connection request arrives. For the backup path

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to be effective, the primary and backup paths should either be link-disjoint (no common links) or node-disjoint (no common nodes).

To improve network performance, the two paths (primary and backup) should be the shortest (i.e. least cost) possible paths. Lightpaths in WDM networks require using the same wavelength along the path, unless wavelength conversion is used. Combining the two constraints (disjoint and shortest) with the wavelength-continuity constraint, the problem of finding the two shortest disjoint paths becomes an NP-Complete problem [3].

Since link failures are more common than node failures, this paper focuses on the case of link-disjoint paths. We also assume no wavelength conversion, as wavelength conversion hardware is generally too expensive to be installed at each node [4]. The goal of this paper is to present an algorithm for finding the two shortest link-disjoint paths in WDM networks. The algorithm presented in this paper takes a new approach for addressing this problem by making it equivalent to the single shortest path problem on another network topology called *auxiliary network topology*. This allows using ordinary shortest path algorithms to solve this problem. There are some constraints that must be considered when applying the shortest path algorithm to the auxiliary network topology to ensure that the two paths are actually link-disjoint. In this paper we base our method on Dijkstra algorithm and show the modifications required for it to work on the auxiliary network topology.

The remaining of this paper is organized as follows. Section 2 provides literature review and related work on path recovery algorithms for WDM networks. In Sect. 3, we describe how to construct the auxiliary network topology. Following that, the details of using modified shortest path algorithm and an example demonstrating the algorithm operation are presented in Sect. 4. To prove the effectiveness of our method, we compare it with two well-known heuristics and with the optimal solution using integer linear programming (ILP). To do this, we develop the ILP formulation of the problem and apply the three methods on the ARPANET network and the STC optical backbone network, under multiple network usage scenarios. This is done in Sect. 5. Section 6 provides an analytical study the performance of the presented method. Finally, Sect. 7 concludes the paper by providing a summary and future work.

## 2 Related Work

The topic of optical network survivability has been addressed extensively in the literature. Techniques for both protection and restoration have been published and analyzed. In this paper, we focus on protection techniques, which involve finding two optimal disjoint paths prior to connection setup. The

simplest method for finding the two shortest paths is known as the two-step algorithm. In the first step of this algorithm, the shortest path is calculated using Dijkstra algorithm. Then, the links used in the shortest path are eliminated from the network. The second step applies Dijkstra to the modified network to find the second shortest path. Despite its simplicity, this algorithm does not work for many network topologies. In some “trap topologies”, the first shortest path splits the network in such a way that no other paths between the two end points can be found, although those paths were available in the original network [5]. A more efficient algorithm for finding optimal disjoint paths was developed by Suurballe [6] and was later improved by Suurballe and Tarjan [7]. This algorithm also works in two steps. In the first step, Dijkstra is applied to the original network to get the first shortest path. In the second step, the links which belong to the first shortest path are reversed and assigned negative costs. Then, Dijkstra is applied to the modified network. The links common between the paths of step 1 and step 2 are eliminated, yielding the two shortest paths between the two end nodes. The Suurballe algorithm is guaranteed to find in polynomial time the two shortest disjoint paths if they exist, provided that the same wavelength is used in both paths [8,9]. However, Suurballe algorithm was not designed to handle cases where the two disjoint paths have to use different wavelengths.

Although the problem of finding the two shortest disjoint paths with the wavelength-continuity constraint has long been believed to be NP-complete, the first formal proof was provided in [8]. Moreover, another study has shown that the problem is NP complete even without the length constraint [3]. The optimal solution for this problem can be obtained using integer linear programming (ILP). ILP Problem formulations for the problem have been developed in [3,8,9]. However, the execution time for ILP is exponential in network size, which prohibits using such a method for moderate to large scale networks. Therefore, most of the studies that addressed this problem have focused on developing heuristic algorithms. In [8], two simple algorithms are proposed. The first algorithm, APF (Active Path First), is essentially the two-step algorithm mentioned earlier. It attempts to find the first shortest path. It then removes the channels used in that path before attempting to find the second path on the modified network. If either attempt fails, the call request is blocked. The second algorithm, APF Enhanced (APFE), improves the APF by reducing the number of shared links between the two paths. In [9], two heuristic solutions are proposed. The first one, Route-First, scans all fiber links and increases the cost of each link linearly with the number of wavelengths already in use. It then runs Suurballe algorithm and checks the two returned routes. If each route has at least one wavelength available in its links, the algorithm will succeed. Otherwise, the algorithm will fail. The second algorithm, Wavelength-Scan, scans each wavelength for two link-disjoint paths using