A tabu search heuristic for the single vehicle pickup and delivery problem with time windows

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The single vehicle pickup and delivery problem with time windows is a generalization of the traveling salesman problem. In such a problem, a number of transportation requests have to be satisfied by one vehicle, each request having constraints to respect: a pickup at its origin and a delivery at its destination, and a time window at each location. The capacity of the vehicle has to be respected. The aim is to minimize the total distance traveled by the vehicle. A number of exact and approximate solution methods exists in the literature, but to the author’s knowledge none of them make use of metaheuristics, still promising with other vehicle routing problems. In this paper we present tabu search and probabilistic tabu search. Results obtained on classical traveling salesman problems and a class of randomly generated instances indicate that our approach often produces optimal solutions in a relatively short execution time.

Keywords: Pickup and delivery, time windows, vehicle routing, tabu search, metaheuristics

1. Introduction

The single vehicle pickup and delivery problem with time windows (1-PDPTW) implies a single vehicle that has to satisfy transportation requests issued by customers, each request requiring a fixed quantity to be picked up at an origin and to be delivered at a destination. A set of constraints has to be respected: capacity of the vehicle, precedence between pickup and delivery locations, time windows at each stop route. The objective is to construct a route through the locations of the requests, such that all side constraints are respected and total costs involved in the execution of the route are minimized. Such problems appear in a wide variety of distribution systems. In special cases where customer requests concern transportation of people, the problem is called dial-a-ride problem with time windows (DARPTW). It mainly arises in transportation systems for the handicapped and the elderly. In these situations, the time constraints imposed by the customers strongly restrict the total vehicle load at any point in time, and the capacity constraints are of second importance.

With a graph theoretical perspective, the single vehicle pickup and delivery problem with time windows can be stated as follows. Let $G = (V,E)$ be a complete non-oriented graph, where $V = \{v_i | i = 0, \ldots, 2n\}$ is the set of vertices and $E = \{(v_i, v_j) | v_i, v_j \in V, i, j = 0, \ldots, 2n, i < j\}$ is the set of edges. Simplifying notations without loss of generality, we can define $V^+ = \{v_1, \ldots, v_n\}$ as the set of pickup locations and $V^- = \{v_{n+1}, \ldots, v_{2n}\}$ as the set of delivery locations, whereas $v_0$ is the depot vertex for the vehicle. We thus obtain $V = V^+ \cup V^- \cup \{v_0\}$.

A set of transportation requests $R = \{r_i : i = 1, \ldots, n\}$ has to be satisfied. A transportation request $r_i$ issued by a customer is characterized by the following.
- A fixed non-negative quantity \( q_i \) to be picked up at vertex \( v_i \) and to be delivered at vertex \( v_{n+i} \) (where \( q_0 = 0 \) at vertex \( v_0 \)).
- A pickup time service \( s_i \) at location \( v_i \) and a delivery time service \( s_{n+i} \) at location \( v_{n+i} \) (where \( s_0 = 0 \) at vertex \( v_0 \)).
- Time windows \([e_i, t_i]\) and \([e_{n+i}, t_{n+i}]\) for pickup and delivery locations respectively, where \( e_i \) and \( t_i \) are the lower and upper bounds of the time windows, in which the service has to be provided.

Each edge \((v_i, v_j)\) from \( E \) is weighted by a cost distance \( c_{ij} \) and a travel time \( t_{ij} \) between vertices \( v_i \) and \( v_j \). We consider in this case the symmetric version of the (1-PDPTW), so we have \( c_{ij} = c_{ji} \) and \( t_{ij} = t_{ji} \) for \( i = 0, \ldots, 2n \). The associated matrices verify the triangular inequalities.

A single vehicle of fixed capacity \( C \) is available to satisfy the transportation requests. A time window \([e_0, t_0]\) is specified for the depot, corresponding to the time available for the vehicle to perform all the requests. The route must be found in order to satisfy the following constraints:

- The vehicle starts from and finishes its route to the depot \( v_0 \).
- Each transportation request \( r_i (i = 1, \ldots, n) \) is satisfied.
- Each location \( v_i \in V^+ \cup V^- \) is visited exactly once.
- The vehicle capacity \( C \) can never be overloaded during the route.
- For each transportation request \( r_i \in R \), the pickup location \( v_i \) is visited before the delivery location \( v_{n+i} \).
- For each vertex \( v_i \in V \), the service begins within the time window \([e_i, t_i]\).

The objective function is to minimize the total distance traveled by the vehicle. As can be seen, the problem is the static version of the single vehicle pickup and delivery problem with hard time windows:

- The upper bound \( t_i \) of a location \( v_i \in V \) cannot be violated. More precisely, the beginning of the service must be provided within the time window. If the vehicle arrives before the lower bound \( e_i \), it must wait until \( e_i \) to start its service. On no account can it not arrive on the location after the upper bound \( t_i \), but it may leave the node after \( t_i \) if the service time is too long.

- All transportation requests are known in advance. Dynamic aspects of the problem, as for instance the dynamic arrival of requests during execution of the route, are beyond the scope of this paper. See Bodin et al. (1983) for further details on real-time routing problems.

According to the above definitions, the reader can easily see the (1-PDPTW) as a constrained version of the traveling salesman problem (TSP), transforming it into a \( NP \)-hard problem. Because of strong constraints—capacity of the vehicle, time windows and precedence relationships between the origin and destination of each customer—even finding a feasible solution may be difficult. However, as Savelberg and Sol (1995) specify it, an optimisation method can benefit from the presence of these strong constraints to reduce the solution space.

Remainder of the paper is organized as follows. In the second part, a review of the literature introduces several approaches for the resolution of the (1-PDPTW) and (m-PDPTW). Next, a tabu search heuristic and a probabilistic tabu search heuristic are developed for the (1-PDPTW). Results are then presented, before a conclusion and final remarks.

2. Literature review

Surprisingly few papers have been published on the pickup and delivery problem with time windows, compared to the classical vehicle routing literature. In this paper we will focus on the single vehicle problem version (1-PDPTW), noting works about the (1-PDPTW) may serve as a basis for the (m-PDPTW). Furthermore, we refer the interested reader to Bodin et al. (1983) and Desrochers et al. (1988) for excellent surveys of vehicle-routing problems with a special consideration of pickup and delivery problems. More recently, Savelberg and Sol (1995) present a survey of the problem types and solution methods found in the literature about the general pickup and delivery problem (GPDP).

2.1. Single vehicle

In the literature review, we distinguish between exact and approximate methods in solving pickup and delivery problems. We will begin with the exact methods encountered. Considering the single-vehicle