A Dynamic Frictional Contact Problem with Normal Damped Response

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Abstract. We consider a mathematical model which describes the frictional contact between a viscoelastic body and a reactive foundation. The process is assumed to be dynamic and the contact is modeled with a general normal damped response condition and a local friction law. We present a variational formulation of the problem and prove the existence and uniqueness of the weak solution, using results on evolution equations with monotone operators and a fixed point argument. We then introduce and study a fully discrete numerical approximation scheme of the variational problem, in terms of the velocity variable. The numerical scheme has a unique solution. We derive error estimates under additional regularity assumptions on the data and the solution.

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1. Introduction

We recently investigated a number of problems related to frictional contact involving viscoelastic materials. For instance, a model for bilateral contact with friction was analyzed in [3], a model for frictional contact with normal compliance was analyzed in [9, 21] and the problem of frictional contact with normal damped response was studied in [1, 8, 23]. In these papers the material was assumed to have a viscoelastic constitutive relation of the form

\[ \sigma = A \varepsilon(\dot{u}) + G \varepsilon(u), \]  

(1.1)

where \( u \) denotes the displacement field, \( \sigma \) the stress field, \( \varepsilon(u) \) the linearized strain tensor, \( A \) and \( G \) are given nonlinear constitutive functions, and the dot above a variable represents its time derivative. The results obtained in [1, 3, 8, 21, 23] deal with the existence and uniqueness of weak solutions, i.e. solutions which satisfy variational formulations of the corresponding mechanical problems. In [3, 8, 9], numerical schemes and their error estimates for the models were also studied. In all the above papers only quasistatic processes were considered.
Dynamic contact problems for viscoelastic materials have been studied by many authors. For instance, dynamic contact problems with normal compliance and Coulomb friction have been considered in [10, 16, 17]. In [10] the coefficient of friction was assumed to depend on the slip \( u_\tau \), and the existence of a weak solution is proved by using the Galerkin method. In [17] the coefficient of friction was assumed to depend on the slip velocity \( \dot{u}_\tau \), and the weak solution of the model is obtained using a theory of evolutionary inclusions for set-valued pseudomonotone maps. A similar method was used in [18] in the study of a bilateral frictional contact problem with discontinuous friction coefficient. Existence of the solution to a dynamic viscoelastic contact problem with Coulomb friction was provided in [14] by arguments of penalization and regularization. There, the contact was modeled with unilateral conditions in velocities and the coefficient of friction was assumed to depend on the solution. The analysis of a class of implicit evolutionary variational inequalities with emphasis in the study of dynamic contact problems for viscoelastic materials may be found in [5, 6]. In all the papers quoted above in this paragraph the material behavior was modeled with a Kelvin–Voigt viscoelastic constitutive law of the form (1.1) in which the viscosity operator \( A \) and the elasticity operator \( G \) were assumed to be linear, with the usual assumptions of symmetry and ellipticity.

Dynamic contact problems for viscoelastic materials with a singular memory were studied in [12, 13]. There, mathematical difficulty for the analysis of the problem was caused by the Signorini boundary conditions formulated in displacements and the existence of a weak solution to the problems was obtained using a penalization method and dual estimates.

Now that variational analysis including existence and uniqueness results in the study of various dynamic viscoelastic contact problems has been established, there is a considerable need to provide the numerical analysis of these problems. The aim of this paper is to provide such kind of results in the study of a simplified model, as a first step in the numerical analysis of more realistic physical problems. We use (1.1) as the constitutive law in which \( A \) and \( G \) are supposed to be nonlinear constitutive functions. As in [1], we assume that the normal stress on the contact surface depends on the normal velocity, i.e. it satisfies a so-called normal damped response condition. This condition models the possible behavior of a layer of lubricant on the contact surface. The friction is modeled with the assumption that the tangential shear on the contact surface is given as a function of the tangential velocity, which again models the physical setting when the contact surface is lubricated. Considering dynamic processes for Kelvin–Voigt materials of the form (1.1) with these contact boundary conditions lead to a new mathematical model involving second-order nonlinear evolution equations. We establish the existence of a unique solution to the model using arguments of maximal monotone operators and fixed points. Then we introduce a numerical approximation scheme and derive error estimates under additional regularity assumptions on the data and the solution. We note that, in a variational form, the frictional problem we study here leads to a system of equations. Therefore, the mathematical arguments we use