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Lower Bounds for Scheduling on Identical Parallel Machines with Heads and Tails

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Abstract. In this paper, we investigate new lower bounds for the $P|r_j, q_j|C_{\max}$ scheduling problem. A new bin packing based lower bound, as well as several new lifting procedures are derived for this strongly $\mathcal{NP}$-hard problem. Extensive numerical experiments show that the proposed lower bounds consistently outperform the best existing ones.

Keywords: scheduling, identical parallel machines, release dates, delivery times, makespan, lower bound

In this paper, we investigate lower bounding strategies for the problem of scheduling identical parallel machines with release dates and delivery times. This problem is formally described as follows: A set J of n jobs has to be scheduled on m identical parallel machines (with $n > m \geqslant 2$). Each job j has a release date or *head* r_j, a processing time or *body* p_j and a delivery time or *tail* q_j. All data are assumed to be deterministic and integer. Each machine processes at most one job at one time and each job cannot be processed by more than one machine at one time. We assume that preemption is not allowed and that all machines are available from time zero onwards. Let $t_j \geqslant r_j$ denote the start time of job j, then the completion time of j is defined as $C_j = t_j + p_j + q_j$. The objective is to find a schedule that minimizes the maximum completion time (or *makespan*) $C_{\max} = \max_{j \in J} C_j$. Following Lawler et al. (1993), this problem is denoted by $P|r_j, q_j|C_{\max}$.

The $P|r_j, q_j|C_{\max}$ is $\mathcal{NP}$-hard in the strong sense since it generalizes the well-studied $P||C_{\max}$ which is strongly $\mathcal{NP}$-hard (Garey and Johnson, 1978). However, unlike this latter problem for which there exist very efficient exact algorithms (Dell'Amico and Martello, 1995), computational experiments with optimization algorithms developed by Carlier (1987) and Gharbi and Haouari (2002) provide evidence that only small or medium-sized instances could be solved within a reasonable computing time. The objective of this paper is to derive several new lower bounds for the $P|r_j, q_j|C_{\max}$. These bounds might be useful both to improve the effectiveness of branch and bound algorithms and to benchmark heuristic solutions.

Our motivation for the study of the $P|r_j, q_j|C_{\max}$ stems from the fact that it constitutes a fundamental model encompassing several well-studied parallel machine schedul-

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ing problems, including $P||C_{\max}$, $P|r_j|C_{\max}$, $P||L_{\max}$, and the parallel machine problem with machine availabilities denoted by $P, NC_{\text{inc}}||C_{\max}$ (Schmidt, 2000). Hence, many of the results presented in this paper might be applicable for all these problems. Also, the $P|r_j, q_j|C_{\max}$ has many interesting theoretical applications. In particular, it arises as a strong relaxation of the Multiprocessor Flow Shop problem (Hoogeveen et al., 1995; Perregaard, 1995; Vandevelde, 1994) and it plays a central role in some exact algorithms for the Resource Constrained Project Scheduling Problem (Carlier and Latapie, 1991).

The paper is organized as follows: In section 2, we survey lower bounds from the literature. In section 3, a new lower bound based on the bin packing problem is proposed. In sections 4–6 several strategies aimed at enhancing lower bounds are introduced. These strategies are based on considering subset of jobs, machines, or both. In section 7, we present the results of an extensive computational analysis of the different lower bounds. Finally, we conclude by providing a summary of our results and indicating some directions for future research.

For convenience, we assume hereafter that the release dates, processing times, and delivery times are sorted according to the non-decreasing order. The values $\bar{r}_j(J)$, $\bar{p}_j(J)$ and $\bar{q}_j(J)$ denote the jth smallest release date, processing time and delivery time in J, respectively. We denote by $\lceil x \rceil$ the smallest integer that is larger than or equal to x, and by $\lfloor x \rfloor$ the largest integer that is smaller than or equal to x.

1. Lower bounds from the literature

1.1. Simple lower bounds

Obviously, no job j can finish its processing earlier than $r_j + p_j + q_j$. Thus, a simple lower bound is:

$$LB_0(J) = \max_{j \in J}(r_j + p_j + q_j),$$

LB_0 can be computed in $O(n)$ time. A second simple $O(n)$ lower bound given by Carlier (1987) is:

$$LB_1(J) = \min_{j \in J} r_j + \left\lceil \frac{1}{m} \sum_{j \in J} p_j \right\rceil + \min_{j \in J} q_j.$$

This bound is based on the fact that all release dates and delivery times are assumed to be equal to $\min_{j \in J} r_j$ and $\min_{j \in J} q_j$, respectively.

Now, in any feasible schedule, there is one machine which is necessarily idle from time zero to $\overline{r}_1(J)$, a second one from time zero to $\overline{r}_2(J), \ldots$, and an mth one from time zero to $\overline{r}_m(J)$. Similarly, there is one machine which is idle from time $C^*_{\max}(J) - \overline{q}_1(J)$ to $C^*_{\max}(J), \ldots$, and an mth one from time $C^*_{\max}(J) - \overline{q}_m(J)$ to $C^*_{\max}(J)$. Moreover, the