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Relativistic Effects and Solar Oblateness from Radar Observations of Planets and Spacecraft

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Abstract—We used more than 250 000 high-precision American and Russian radar observations of the inner planets and spacecraft obtained in the period 1961–2003 to test the relativistic parameters and to estimate the solar oblateness. Our analysis of the observations was based on the EPM ephemerides of the Institute of Applied Astronomy, Russian Academy of Sciences, constructed by the simultaneous numerical integration of the equations of motion for the nine major planets, the Sun, and the Moon in the post-Newtonian approximation. The gravitational noise introduced by asteroids into the orbits of the inner planets was reduced significantly by including 301 large asteroids and the perturbations from the massive ring of small asteroids in the simultaneous integration of the equations of motion. Since the post-Newtonian parameters and the solar oblateness produce various secular and periodic effects in the orbital elements of all planets, these were estimated from the simultaneous solution: the post-Newtonian parameters are $\beta = 1.0000 \pm 0.0001$ and $\gamma = 0.9999 \pm 0.0002$, the gravitational quadrupole moment of the Sun is $J_2 = (1.9 \pm 0.3) \times 10^{-7}$, and the variation of the gravitational constant is $\dot{G}/G = (-2 \pm 5) \times 10^{-14}\ \text{yr}^{-1}$. The results obtained show a remarkable correspondence of the planetary motions and the propagation of light to General Relativity and narrow significantly the range of possible values for alternative theories of gravitation.

Key words: *celestial mechanics, cosmology, Sun.*

INTRODUCTION

Radar observations of planets began in 1961 and have been widely used in astronomical practice ever since. High-precision radar measurements spanning a time interval of more than forty years allow not only the orbital elements of the planets, but also other constants of the planetary theory, including the relativistic parameters, to be determined with a high accuracy.

Of the three main tests of General Relativity in the Solar system (the secular motions of the planetary perihelia, the signal delay, and the deflection of light in a gravitational field), the first two tests have been performed using radar observations of planets and spacecraft.

The main and best determined relativistic effect in the Solar system is the secular motion of Mercury's perihelion that was discovered by Le Verrier in 1859. For him, this was a major problem of the discrepancy between theoretical predictions and observations, and it was explained in 1915 by Einstein's theory of General Relativity. However, the secular motion of Mercury's perihelion is known to depend on a linear combination of the post-Newtonian parameters (β, γ) and the gravitational quadrupole moment of the Sun (J_2). Papers (see, e.g., Pireaux and Rozelot 2003) arguing that only this combination rather than the three parameters themselves could be determined from current observations have appeared in recent years. However, the post-Newtonian parameters and the solar oblateness cause different secular and periodic perturbations both for different orbital elements (and not just for the perihelia) and for different planets. In addition, the parameter γ can also be determined from Shapiro's effect, which allows all three parameters to be estimated. Since these parameters can in most cases be obtained by analyzing the secular variations of orbital elements, the errors of their determination decrease with increasing time interval of observations. At the same time, the errors in the secular variation of the gravitational constant ($\dot{G}/G$), one of the most interesting parameters, decrease even faster: as the square of the time interval. This allows $\dot{G}/G$ to be estimated, thereby basically verifying the strong equivalence principle, since many theories of gravitation predict a variation of the locally measured Newtonian gravitational constant with time on the evolutionary scale of the Universe.

Some of the recent post-Newtonian-parameter

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determinations, e.g., $\gamma = 1.000021 \pm 0.000023$, from Cassini radar observations (Bertotti *et al.* 2003) reach a high accuracy. However, the improvement in quality and the increase in the number of current radar observations of planets and spacecraft as well as the increase in the time interval of observations have allowed not only γ, but also β, $\dot{G}/G$, and the gravitational quadrupole moment of the Sun to be estimated independently and from other data.

THE METHOD, EPM EPHEMERIDES

We used the following method to calculate the relativistic parameters and the solar oblateness. First, we constructed a numerical theory for the motion of the planets and the Moon, EPM2004—Ephemerides of Planets and the Moon (Pitjeva 2004, 2005), by using more than 317 000 observations (1913–2003) of various types. These included radiometric measurements of planets and spacecraft, astrometric CCD observations of the outer planets and their satellites, and meridian and photographic observations. Apart from the planetary ephemerides, we also constructed the ephemerides of the orbital and rotational motion of the Moon that were improved by processing the 1970–2003 LLR observations (Krasinsky 2002). The ephemerides of the planets and the Moon were constructed by the simultaneous numerical integration of the equations of motion for all planets, the Sun, the Moon, 301 largest asteroids, rotation of the Earth and the Moon, including the perturbations from the solar oblateness and the asteroid ring that lies in the plane of the ecliptic and consists of the remaining smaller asteroids. The equations of motion for bodies were taken in the post-Newtonian approximation in the Schwarzschild gravitational field described by a three-parameter (α, β, γ) metric in a harmonic coordinate system with $\alpha = 0$; all versions of the ephemerides were constructed for General Relativity: $\beta = \gamma = 1$. The general equations of motion for bodies in a nonrotating barycentric coordinate system are

$$\ddot{\mathbf{r}}_i = A + B + C + D,$$

where A are the Newtonian gravitational accelerations, B are the relativistic terms (Newhall *et al.* 1983), C are the terms attributable to the solar oblateness, and D are the terms attributable to the asteroid ring (Krasinsky *et al.* 2002).

Below, we provide brief information about the EPM2004 theory and its construction (Pitjeva 2005).

First, a physical model that includes all of the significant factors and that adequately reflects the actual planetary motions underlies this theory. In particular, including the perturbations from the several largest asteroids, as was done in previous versions of our EPM or Jet Propulsion Laboratory (JPL) DE ephemerides, was shown (Krasinsky *et al.* 2001; Standish and Fienga 2002) to be insufficient. In EPM2004, the gravitational perturbations that are introduced into the orbits of the inner planets by asteroids and that make it difficult to determine the parameters were reduced significantly by including 301 large asteroids and the perturbations from the massive ring of small asteroids in the simultaneous integration of the equations of motion and by estimating their masses when processing the observations.

Second, the accuracy of the numerical integration itself was checked by comparing the results of the forth and back integrations on a hundred-year time interval. The emerging errors were at least an order of magnitude smaller than the observational errors. Thus, the accuracy of the ephemerides is determined mainly by the accuracy of the observations and their reductions.

Third, producing the ephemerides is an iterative process of comparing the constructed ephemerides with observations, improving the parameters by the least-squares method (LSM), introducing these in the theory, and constructing a new version of the ephemerides.

In the main improvement of the planetary part of the EPM2004 ephemerides, we determined about 200 parameters: the orbital elements of all planets and the 13 satellites of the outer planets the observations of which were used to improve the orbits of these planets; the astronomical unit in kilometers; three orientation angles of the ephemerides relative to the International Celestial Reference Frame (ICRF); the rotation parameters of Mars (two orientation angles of the equator of Mars relative to its orbit and their secular variations, the velocity, and eight coefficients of the seasonal rotation terms of the Martian axis) and the coordinates of three landers on the Martian surface; the masses of the bodies (Jupiter and the six asteroids that perturb Mars most strongly), the mean densities for three taxonomic classes of asteroids (C, S, M), the mass and radius of the asteroid ring, the ratio of the Earth's and Moon's masses; the gravitational quadrupole moment of the Sun (J_2) and twelve parameters of the solar corona for different conjunctions with the Sun; eight coefficients of Mercury's topography and the corrections to the level surfaces of Venus and Mars relative to which the topographies of these planets were calculated; five parameters for calculating the additional phase effect in the optical observations of the outer planets; and the constant shifts for six groups of observations that were interpreted as systematic errors or calibration errors of the instrumentation.

Once the EPM2004 ephemerides were constructed from all radar observations of the inner planets, spacecraft passing by or orbiting these planets, and