Optimization of State-linear Discrete-Continuous Systems

I. V. Rasina* and O. V. Baturina**

* Siberian Academy of Law, Economics, and Management, Irkutsk, Russia
** Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia

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Abstract—An iterative method for process optimization in the state-linear discrete-continuous systems was proposed on the basis of the sufficiency conditions and the minimax scheme of nonlocal improvement. For the discrete systems, in particular, it was reduced to an essentially new method. Consideration was given to the possibility of applying it to the general nonlinear systems through approximation by systems of the class under consideration. Examples were presented.

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1. INTRODUCTION

The interest in the study and optimization of the heterogeneous systems has recently grown appreciably. It is due, on the one hand, to the practical needs for automation in various industries, and, on the other hand, to the explosive progress of the computer engineering and informatics, which is evidenced, in particular, by the avalanche of publications considering heterogeneous differential systems that are most frequently are referred to as the hybrid systems. In this connection, the important advances of the national scientists in the past years [1–5] became topical, and the need for their continuation and development is self-evident. They include the approach based on the hierarchical representation of such systems with the aim of decomposing them into homogeneous systems and using, therefore, the rich arsenal of the methods for the homogeneous systems with proper generalizations and improvements. The sufficient general optimality conditions of the Krotov type and the iterative methods of optimization for the nonlinear two-level discrete-continuous systems (DCS) of the general form are given in [6, 7].

The present paper proposes an iterative method of process optimization in the state-linear discrete-continuous systems that was developed on the basis of sufficient conditions and the minimax scheme of nonlocal improvement. Consideration is given to the possibility of applying DCS and this method to the nonlinear systems through approximations by systems of the class under consideration.

2. FORMULATION OF THE PROBLEM

Consideration is given to a special case of the DCS model [7]:

$$x(k+1) = A(k,u)x(k) + b(k,u),$$

$$x \in \mathbb{R}^{m(k)}, \quad u \in U(k) \subset \mathbb{R}^{p(k)}, \quad k \in K = \{k_I, k_I + 1, \ldots, k_F\},$$

$$\dot{x}^c = A^c(k,z,t,u^c)x^c + b^c(k,z,t,u^c),$$

$$t \in T(z), \quad T = [t_I, t_F], \quad k \in K' \subset K, \quad k_F \notin K',$$

$$x^c \in \mathbb{R}^{n(k)}, \quad u^c \in U^c(k,t) \subset \mathbb{R}^{q(k)}, \quad u^d \in U^d(k) \subset \mathbb{R}^{r(k)}, \quad z = (x, u^d).$$
In the right side of (1), the operator at \( k \in \mathbf{K}^\prime \) comes to \( \alpha(k, z)\gamma^c + \beta(k, z) \), where
\[
\gamma^c = (t_I, x^c_I, t_F, x^c_F) \in \Gamma^c(z) = \{ \gamma^c : t_I = \tau(z), \quad x^c_I = \xi(z), \quad t_F = \vartheta(z), \quad x^c_F \in \mathbb{R}^m(k) \}.
\]
Here, \( k \) is the number of the step (stage), not necessarily the physical time, \( U(k) \) and \( U^d(k) \) are the given sets, \( \tau(z), \xi(z), \) and \( \vartheta(z) \) are the given functions, and \( z = (x, u^d) \) is the collection of the variables of the discrete chain (upper level) playing the role of parameters. Here and below, all dependencies on \( z \) are regarded as linear:
\[
\Lambda(z, q) = \Lambda_0(q) + \Lambda_z(q)z,
\]
where \( q \) is the collection of the rest of arguments. They are not included for brevity.

By the discrete-continuous process is meant the collection \( m = (x(k), u(k)) \), where \( m^c \) is a continuous process \( (x^c(k, t), u^c(k, t)) \), \( t \in T(z) \) for \( k \in \mathbf{K}^\prime \): \( u(k) = (u^d(k), m^c(k)) \).

We denote by \( \mathbf{D} \) the set of permissible processes satisfying the aforementioned conditions under fixed \( k_I = 0, k_F, \) and \( x(k_I) \), sectionally continuous \( u^c(t) \) and sectionally smooth \( x^c(t) \). The corresponding set of continuous processes \( m^c \) is denoted by \( \mathbf{D}^c(z) \) for each \( z \).

For model (1) and (2) we consider the standard problem of optimal control as that of minimum of the functional \( I = c^T x(k_F) + d \) on \( \mathbf{D} \) where \( c \) is the \( n(k_F) \)-vector. We notice that the constant \( d \) does not affect the solution and can be defined arbitrarily from formal considerations.

### 3. ITERATIVE ALGORITHM

The formulated problem is solved by constructing an iterative procedure on the basis of the sufficient improvement conditions and the minimax Krotov principle, an approach similar to [8, 9]. Linear-in-variables states of the functions \( \varphi(k, x) = \psi^T(k)x \) and \( \varphi^c(z, t, x, x^c) = \psi^c^T(k, z, t)x^c \) are introduced, where \( \psi(k) \) is arbitrary and \( \psi^c(z, t) \) is sectionally smooth in \( t \). The Lagrangian
\[
L = G(x(k_F)) - \sum_{k \in \mathbf{K} \setminus k_F} R(k, x(k), u(k))
\]
\[
+ \sum_{k \in \mathbf{K}^\prime} \left( G^c(z(k)) - \int_{T(z(k))} R^c(z(k), t, x^c(t), u^c(t))dt \right),
\]
\[
G(x) = c^T x + d + \psi^T(k_F) - \psi^T(k_I)x_I,
\]
\[
R(k, x, u) = H - \psi^T(k)x,
\]
where \( H = \psi^T(k + 1)(A(k, u)x(k) + b(k, u)) \),
\[
G^c(k, z, \gamma^c) = -H(k, z, \gamma^c) + \psi^T(k)x + \psi^c^T(k, z, t_F)x^c_F - \psi^c^T(z, t_I)x^c(t_I),
\]
\[
t_I = \tau(z), \quad t_F = \vartheta(z), \quad x^c(t_I) = \xi(z),
\]
where \( H = \psi^T(k + 1)(\alpha(k, z)\gamma^c + \beta(k, z)) \),
\[
R^c(k, z, x, \gamma^c) = H^c + \psi^c^T(z, t)x^c,
\]
where \( H^c(k, z, t, u^c, x^c) = \psi^c^T(z, t)(A^c(k, z, t, u^c)x^c + b^c(k, z, t, u^c)) \), is constructed as a specification of the generalized Lagrangian in [7]. This functional is considered on the extension \( E \) of the set \( \mathbf{D} \) at the expense of eliminating the discrete and differential links. For the aforementioned links, \( L(m) = I(m) \) according to the principle of extensions [10].

The following assertions are obvious: