Monte Carlo Simulation of Angular Characteristics for Polarized Radiation in Water-Drop and Crystal Clouds

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Received December 20, 2009

DOI: 10.1134/S1024856010050040

Abstract — In the paper we present the results of computational experiments aimed to define the angular distributions for the polarized radiation scattered in a cloudy layer. The angular distributions for Stokes parameters, degree of polarization scattered upward and downward by water-drop and crystal clouds. The ulterior objective of the research is to develop effective techniques to study the particles shape and size by measuring angular characteristics of the scattered radiation emanating from clouds.

INTRODUCTION

The role of clouds in the global climate system is important but not well studied. Cloud feedbacks are the largest source of uncertainty in estimates of radiation balance and climate sensitivity, therefore a better understanding and representation of radiation transfer processes in clouds is of paramount importance for climate science. On the other hand, the adequate optical models of clouds are necessary to investigate properties of cloudiness by active and passive optical remote sensing. Our paper deals with numerical modeling of the solar radiation transfer in the atmosphere clouds taking into account specific features caused by polarization of light. By computational experiments we studied angular distributions for the polarized radiation scattered upward and downward by water-drop and crystal clouds. The angular distributions were computed for the Stokes parameters, degree of polarization, and direction of preferable polarization. For computations we used Monte Carlo method and several optical models of clouds. The ultimate aim of the research is to develop effective techniques to study phase structure of clouds, shape and size of particles by measuring characteristics of the scattered radiation.

1. MATHEMATICAL MODEL AND MONTE CARLO ALGORITHMS

Assume that an optically isotropic scattering medium consists of particles randomly oriented in space, extinction coefficient and single scattering albedo in the medium does not depend on light polarization, and a field of reference-vectors \( \rho(\omega) \) is fixed, i.e. for every direction \( \omega \in \Omega = \{ (a, b, c) \in \mathbb{R}^3 : a^2 + b^2 + c^2 = 1 \} \) there is defined a unit vector \( \rho(\omega) \) orthogonal to \( \omega \). Then the process of stationary polarized radiation transfer in the scattering medium may be described by integral equations of the second kind with the generalized kernel

\[
S[\rho](r, \omega) = \int_{\Omega} \int_{\mathbb{R}^3} \frac{e^{-\tau(r', r)}}{|r - r'|^2} q(r') \sigma(r') M[\rho', \rho](\omega', \omega, r') \times S'[\rho'](r', \omega') \delta\left(\omega - \frac{r - r'}{|r - r'|^2}\right) dr'd\omega' + S_0[\rho](r, \omega),
\]

(1)

\( r', r \in \mathbb{R}^3 \), \( \omega', \omega \in \Omega \), \( \rho = \rho(\omega) \), \( \rho' = \rho(\omega') \). Here \( S[\rho](r, \omega) \) is the Stokes vector (we shall consider Stokes vectors of the type \((I, Q, U, V)\)) with respect to the reference vector \( \rho = \rho(\omega) \) for the radiation at the point \( r \) spreading in the direction \( \omega \), \( q(r') \) is the single scattering albedo at the point \( r' \), \( \sigma(r') \) is the extinction coefficient at the point \( r' \), \( \delta \) is the delta-function, \( S_0[\rho](r, \omega) \) is the Stokes vector of the source at the point \( r \) in the direction \( \omega \), \( \tau(r', r) = \int_0^l \sigma(r(s), \omega) ds \) is the optical length of the segment \([r', r] \), \( l = ||r - r'|| \), \( M[\rho', \rho](\omega', \omega, r') \) is the 4 \( \times \) 4-phase matrix of the medium at the point \( r' \) \( (\omega' \) is the direction before scattering, and \( \omega \) is the direction after scattering):

\[
M[\rho', \rho](\omega', \omega, r') = L[\rho, \rho^*]^{-1} M(\omega', \omega, r') L[\rho', \rho^*],
\]

where \( M(\omega', \omega, r') \) is the Mueller matrix with normalization for the first element \( \int_{\Omega} m_{11}(\omega', \omega, r') d\omega = 1 \), \( L[\rho, \rho^*] \) is the rotation matrix corresponding to

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\textsuperscript{1} The article is published in the original.

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exchance of reference vectors: \( S[\rho^*] = L[\rho, \rho^*]S[\rho] \)

and \( \rho^* \) is the reference vector orthogonal to the plane of scattering \((\omega', \omega)\). Assume that \( \rho^* \) can be obtained by rotation of reference vector \( \rho = \rho(\omega) \) at angle \( \varphi \) around \( \omega \) (in the right-handed coordinate system). The rotation matrix has the form

\[
L[\rho(\omega), \rho^*] = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & a & b & 0 \\
0 & b & -a & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
\]

\( a = \cos 2\varphi, \ b = \sin 2\varphi; \)

\( \rho^* = \frac{\omega' \times \omega}{||\omega' \times \omega||} \cos \varphi = \langle \rho, \rho^* \rangle, \ \sin \varphi = |\rho \rho^* \omega|. \)

Here symbol \( \times \) denotes the vector product, \( \langle ... \rangle \) is the scalar product, and \( [abc] = \langle a, (b \times c) \rangle \) is the mixed product of vectors \( a, b, c \).

Remark. About the polarized radiation transfer equation see, for example, [1–7]. The integral form (1) of the radiation transfer equation is presented in [2] (usually reference-vectors are assumed to be orthogonal to the vertical planes, which contain the direction vectors). In papers [6, 7] the radiation transfer equation is derived as the Kolmogorov backward equation for a specific time continuous stationary Markov jump process.

Under made assumptions about the optical medium, the elements of the Mueller matrix depend only on cosine of the angle between \( \omega' \) and \( \omega \):

\[
M(\omega', \omega, r')d\omega = M(\mu, \psi, r')d\mu d\psi,
\]

\( \mu = \langle \omega', \omega \rangle, \)

where \( \mu \) is the cosine of the angle between direction before and after scattering, and \( \psi \) is the azimuthal scattering angle, i.e. an angle between the scattering plane and a fixed plane containing \( \omega' \). (Note, that in (2) for notation simplicity we use the same letter \( M \) for different mathematical objects.) Moreover, for crystal clouds the Mueller matrix has the form (see, for example, [8, 9])

\[
M(\mu, \psi, r') = \frac{1}{2\pi} \begin{pmatrix}
m_{11} & m_{12} & 0 & 0 \\
m_{12} & m_{22} & 0 & 0 \\
0 & 0 & m_{33} & m_{34} \\
0 & 0 & -m_{34} & m_{44}
\end{pmatrix},
\]

where \( m_{ij} = m_{ji}(\mu, r') \) and the first element \( m_{11} \) can be interpreted as the phase function in case the beam before scattering is unpolarized,

\[
\int m_{11}(\mu, r')d\mu = 1.
\]

For water-drop clouds with spherical particles the Mueller matrix (3) satisfies \( m_{22} = m_{11}, \ m_{44} = m_{33} \).

One of the most effective methods to simulate processes of the polarized radiation transfer in a scattering medium is the Monte Carlo method. The main idea of the Monte Carlo method is to estimate characteristics of radiation fields by computer simulation of a huge number of random photons trajectories in the medium. Several realizations of Monte Carlo approach were developed to solve the polarized radiation transfer equation (1) (see, for example, [2, 6, 7, 10–15]). Below we present a brief description of stochastic algorithms that we used for numerical experiments.

By a “photon” we shall call a quasi-monochromatic wave that will be described by the Stokes vector \( S[\rho](r, \omega) \) with respect to the reference-vector \( \rho \), where \( \rho \) is the vector of the photon’s coordinates in space and \( \omega \) is the unit vector of its movement direction (the reference-vector \( \rho \) is always orthogonal to \( \omega \)). A trajectory of a photon in scattering medium was simulated in following steps.

**Step 1.** Initial point \( r_0 = (\omega_0, y_0, z_0) \), initial direction \( \omega_0 = (a_0, b_0, c_0), \ |\omega_0| = 1 \), and Stokes vector \( S[\rho_0](r_0, \omega_0) \) of a photon are simulated according to the density of sources; \( n = 0 \).

**Step 2.** The photon’s free-path length \( l \) is simulated according to the probability density

\[
p(l) = \sigma(x_n + t\omega_n) \exp\left[-\frac{1}{2} \int_0^l \sigma(x_n + t\omega_n)dt\right], \quad l > 0,
\]

where \( \sigma \) is the extinction coefficient.

**Step 3.** We set \( n = n + 1 \) and calculate the coordinates of the next collision point in the medium:

\[
x_n = x_{n-1} + a_{n-1}l, \quad y_n = y_{n-1} + b_{n-1}l, \quad z_n = z_{n-1} + c_{n-1}l,
\]

\( r_n = (x_n, y_n, z_n) \).

**Step 4.** The scattering of the photon is simulated: a new direction \( \omega_n \) of the photon is simulated according to a scattering function \( g(\omega_{n-1}, \omega_n, r_n) \) and a new value of the Stokes vector is calculated according to the formula

\[
S[\rho_n](r_n, \omega_n) = q(r_n)g^{-1}(\omega_{n-1}, \omega_n, r_n)M(\omega_{n-1}, \omega_n, r_n) \times L[\rho_{n-1}, \rho_n]S[\rho_{n-1}](r_{n-1}, \omega_{n-1}),
\]

where new reference-vector \( \rho_n \) is orthogonal to the scattering plane,

\[
\rho_n = \frac{\omega_n - \omega_{n-1} \times \omega_n}{||\omega_n - \omega_{n-1} \times \omega_n||},
\]

and the scattering function \( g \) can be chosen in different ways as follows. An algorithmically simpler variant that is often used in stochastic algorithms that are used for numerical experiments. An algorithmically simpler variant that is often used in stochastic algorithms that is used for numerical experiments.

Another possibility is to use a physically adequate scattering function for which (4) should give

\[
I(r_n, \omega_n) = q(r_n)I(r_{n-1}, \omega_{n-1}).
\]