1. INTRODUCTION

The study of multiple scattering of light in liquid crystals (LCs) has been attracting much attention for many years [1–9]. The most detailed analysis has been devoted to nematic LCs (NLCs). The optical properties of these systems are well known, and, as a rule, all the necessary optical characteristics of these materials have been measured with high precision. From the standpoint of multiple scattering problems, a characteristic feature of LCs is that multiple scattering in them is attributed to the thermal fluctuations of the director, rather than to the fluctuations of individual particles, as in suspensions, or to structural inhomogeneities, as in inhomogeneous solid dielectrics. The description of the scattering properties of a system of particles is a difficult problem even in the case of spherical particles, because this description requires the application of either the Mie formulas (which are extremely sensitive to the particle size) in the case of low concentrations of particles, or the solution of a diffraction problem for a system of particles in the case of high concentrations. An even more complicated problem is the correct description of scattering by structural inhomogeneities in solid dielectrics. In contrast to such objects, the amplitude and the correlation properties of thermal fluctuations in LCs have been well studied both experimentally and theoretically.

The analysis of multiple scattering in NLCs is complicated by the fact that this system is characterized by significant optical anisotropy. Therefore, one cannot describe scattering in the scalar field approximation, as this is done in the case of suspensions, but should take into consideration polarization phenomena.

In the general case, the order in NLCs is described by the angular distribution of molecules. In the case of Rayleigh scattering, one can restrict oneself to a second-rank tensor order parameter [10]. A general analysis of all types of fluctuations of this parameter was first carried out in [11]. It was shown that one can distinguish three types of fluctuations: transverse uniaxial (or fluctuations of the director), transverse biaxial, and longitudinal fluctuations. The strongest fluctuations are the fluctuations of the director. As for the other types of fluctuations, they can be observed in the single-scattering regime only in those geometries where there is no scattering by transverse uniaxial fluctuations [12]. In the multiple scattering regime, due to chaoticization, all types of fluctuations contribute to the scattering. Therefore, we will restrict the consideration to the strongest fluctuations, namely, to the fluctuations of the director.

One of the most interesting phenomena associated with multiple scattering is coherent backscattering. This phenomenon has been analyzed in detail theoretically and experimentally for various systems [13–15], including LCs [2, 3, 7, 8, 16]. The description of the backscattering peak reduces to the summation of ladder and cyclic diagrams. This problem has been solved precisely for a system of point scatterers [17]; for scatterers of finite size or for fluctuations with finite corre-
luation radius, one introduces approximations whose accuracy cannot always be controlled due to the complexity of the problem.

On the other hand, methods of numerical simulation have been developed to date that allow one to avoid many difficulties arising in analytical calculations. In fact, numerical simulation can be considered as modeling of a real experiment. However, this simulation have been developed to date that allow one to simulate with regard to the real parameters of LCs without making simplifying assumptions. This allows us to quantitatively compare the results obtained with the experimental data [7, 8] and with the results of analytical calculations.

The present study is devoted to the numerical simulation of coherent backscattering in NLCs. Similar calculations have been carried out in a number of studies [5, 7, 18]. In these studies, the authors obtained a backscattering peak and analyzed its behavior under different conditions. However, due to the complexity of the problems, the authors made various simplifying assumptions, such as one-constant approximation, independence of the extinction coefficient of the ray propagation direction, etc. Therefore, they could analyze the coherent backscattering phenomenon only at a qualitative level. In the present study, we carry out simulation with regard to the real parameters of LCs without making simplifying assumptions. This allows us to quantitatively compare the results obtained with the experimental data [7, 8] and with the results of analytical calculations [16], in which the real parameters of a medium have also been taken into account.

The paper is organized as follows. In Section 2, we present necessary data on single scattering in NLCs. In Section 3, we describe a simulation of the propagation of photons in NLCs. Expressions for the coherent backscattering peak are obtained in Section 4. Section 5 contains the results of numerical simulation and their comparison with experimental data and with the results of analytical calculations. The main results of the paper are summarized in the Conclusions.

2. SINGLE-SCATTERING INDICATRIX AND EXTINCTION COEFFICIENT

According to their optical properties, NLCs are uniaxial. The direction of preferred orientation is defined by the unit vector \( \mathbf{n} \) of the director. The dielectric tensor is given by

\[
\varepsilon_{\alpha\beta}(r) = \varepsilon_\perp \delta_{\alpha\beta} + \varepsilon_\parallel n_\alpha(r) n_\beta(r),
\]

where \( \varepsilon_\parallel = \varepsilon_i - \varepsilon_\perp, \varepsilon_\perp \) and \( \varepsilon_\perp \) are dielectric constants along and across the director, and \( \delta_{\alpha\beta} \) is the Kronecker delta.

The fluctuations \( \delta \varepsilon_{\alpha\beta}(r) \) of the dielectric tensor due to the fluctuations of the director are represented as

\[
\delta \varepsilon_{\alpha\beta}(r) = \varepsilon_{\alpha\beta}(r) - \varepsilon^0_{\alpha\beta} = \varepsilon_\perp [n_\alpha \delta n_\beta(r) + n_\beta \delta n_\alpha(r)],
\]

where \( \varepsilon^0_{\alpha\beta} = \varepsilon_\perp \delta_{\alpha\beta} + \varepsilon_\parallel n_\alpha n_\beta \) is the equilibrium dielectric tensor, \( \mathbf{n} \) is the mean direction of the director, and \( \delta \mathbf{n} = \mathbf{n} - n^0 \).

The Fourier transform of the correlation function of the dielectric constant fluctuations, \( B_{\alpha\beta\mu\nu}(q) = k_0^4 (\delta \varepsilon_{\alpha\mu}(\delta \varepsilon^*_{\nu\beta})(q)) \), is related to the fluctuations of the director by the relation

\[
B_{\alpha\beta\mu\nu}(q) = k_0^4 \epsilon_0^2 \sum_{j=1}^{2} \langle |\delta n_\mu(q)|^2 \rangle (a_{ij} a_{j\gamma} n_\alpha n_\beta + a_{ij} a_{j\gamma} n_\alpha n_\beta + a_{ij} a_{j\gamma} n_\alpha n_\beta)
\]

(2.3)

Here, for convenience, we introduced the coefficient \( k_0 \), where \( k_0 \) is the wavenumber in vacuum. For every vector \( \mathbf{q} \), we introduced the unit vectors

\[ a_1(\mathbf{q}_\perp) = \mathbf{q}_\perp / q_\perp, \quad a_2(\mathbf{q}_\perp) = \mathbf{n}^0 \times a_1(\mathbf{q}_\perp), \]

where \( \mathbf{q}_\parallel \) and \( \mathbf{q}_\perp \) are components of the vector \( \mathbf{q} \) along and across \( \mathbf{n}^0 \); \( \langle |\delta n_\mu(q)|^2 \rangle \) is the mean square of the Fourier components of the fluctuations of the director [10, 19],

\[
\langle |\delta n_\mu(q)|^2 \rangle = \frac{k_0 T}{K_\parallel q_\perp^2 + K_\perp q_\parallel^2 + \chi_H H}, \quad l = 1, 2, (2.4)
\]

where the angular brackets denote statistical averaging, \( K_j \) are the Frank moduli, \( j = 1, 2, 3 \), \( T \) is temperature, \( k_0 \) is the Boltzmann constant, \( \chi_H \) is the anisotropy of magnetic susceptibility, and \( H \) is the strength of the external dc magnetic field.

In NLCs, the indicatrix of single scattering by the fluctuations of the director is given by the following expression in the general case [20]:

\[
I_s^{(i)} = I^{(i)} \frac{V}{(4\pi)^2 R^2 n^{(i)} \delta^{(i)}} \frac{1}{\cos^3 \delta^{(s)}} \frac{n^{(s)}}{\cos^3 \delta^{(s)}} \times \int_0^{\pi} e^{(s)} \epsilon^{(s)} B_{\alpha\beta\mu\nu}(q) \epsilon^{(i)}(\mu) e^{(i)}(\nu) d\delta^{(i)}
\]

(2.5)

where the indices “i” and “s” indicate incident and scattered waves, respectively; the indices “e” and “p” correspond to extraordinary and ordinary waves, respectively; \( V \) is the scattering volume; \( R \) is the distance from the scattering volume to the point of observation; \( \mathbf{q} = \mathbf{k}^{(i)} - \mathbf{k}^{(s)} \), where \( \mathbf{k}^{(i)} \) and \( \mathbf{k}^{(s)} \) are the wave vectors of the scattered and incident light; \( \epsilon^{(s)} \) and \( \epsilon^{(i)} \) are the polarization vectors of the scattered and incident light; \( \delta^{(s)} \) and \( \delta^{(i)} \) are the angles between the wave vectors of the scattered and incident light and the Poynting vector; \( I^{(i)} = I^{(i)}(\mathbf{s}) \) is the intensity (the modulus of the Poynting vector \( \mathbf{S}^{(i)} \) of the incident light,

\[
I^{(i)} = |\mathbf{S}^{(i)}| = \frac{\epsilon}{4\pi} \mathbf{k}^{(i)} n^{(i)} \cos \delta^{(i)},
\]