Theory of intracavity-frequency-doubled solid-state four-level lasers

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Abstract  The spatial distributions of the pumping, the population inversion density, and the
intracavity photon densities of the fundamental and the second harmonic are taken into account in the
rate equations of the intracavity-frequency-doubled cw lasers. By normalizing the related parameters, it
is shown that the general solution of these space-dependent rate equations is dependent upon three
dimensionless parameters: the pump to laser-mode size ratio, the normalized pump level, and a
parameter written as $\tau_{\text{SHG}}$, which is related to the ability of the nonlinear crystal to convert the
fundamental to the second harmonic. By numerically solving these rate equations, a group of general
curves are obtained to express the relations between the solution and the three dimensionless
parameters. In addition, the optimal pump to laser-mode size ratio and the optimal $\tau_{\text{SHG}}$ are
determined. A comparison with the result obtained under the plane-wave approximation is also given.

Keywords: solid-state lasers, intracavity frequency doubling, laser theory.

Compact and efficient sources of stable, single-longitudinal-and-transverse-mode laser in the
visible spectral region are attractive for numerous scientific and technical applications such as
laser-based metrology, xerography and biotechnology. Laser-diode-pumped intracavity-frequency-
doubled solid-state lasers are good candidates for this kind of laser sources. LD-pumped solid-
state lasers have the advantages of high efficiency, simplicity, compactness, and good stability.
The intracavity frequency doubling takes advantage of the high power density available within the
laser cavity, and the high-quality nonlinear crystals such as KTP, LBO and the quasi-phase-
matched crystals developed in recent years constitute the ideal frequency-doubling crystals for
these lasers. As a result, LD-pumped intracavity-frequency-doubled solid-state lasers have been
a widely investigated research subject in recent years\cite{1-12}.

Rate equations are efficacious tools for analyzing the performance of lasers. The rate equa-
tions for the intracavity-frequency-doubled cw lasers were first given by Smith\textsuperscript{13}. By considering
the coupling of the longitudinal modes, Baer analyzed the large-amplitude fluctuations of a
multiple-longitudinal-mode laser\textsuperscript{14}. Helmfrid et al. presented a theoretical investigation of the
stability of a single-mode laser\textsuperscript{15}. As these rate equations were obtained under the plane-wave
approximation, which assumed that the intracavity photon density and the population inversion
density were uniform within the beam cross section and the beam cross section of the second
harmonic was equal to that of the fundamental, they cannot describe the dependence of the output
power on the pump to laser-mode size ratio. However, by taking into account the spatial
distributions of the pumping and the intracavity photon density in the rate equations of LD-
pumped cw lasers\cite{16-19}, Q-switched lasers\cite{20,21}, and intracavity-frequency-doubled Q-switched
lasers\cite{22}, it has been shown that the pump to laser-mode size ratio is one of the important
parameters that strongly influence the output characteristics. Therefore, for a more accurate
theoretical analysis of the output characteristics of the intracavity-frequency-doubled cw lasers,
particularly the influence of the pump to laser-mode size ratio on the laser characteristics, it is
desirable to consider the spatial distributions of the pumping, the population inversion density,
and the intracavity photon densities of the fundamental and the second harmonic in their rate
equations.

In this paper, the spatial distributions of the pumping, the population inversion density, and
the intracavity photon densities of the fundamental and the second harmonic in the rate equations
are taken into account. By normalizing the related parameters, it is shown that the general
solution of these space-dependent rate equations (i.e. the normalized output second-harmonic
power) is dependent upon three dimensionless parameters: the pump to laser-mode size ratio, the
normalized pump level, and a parameter written as $\eta_{\text{SHG}}$, which is related to the ability of the
nonlinear crystal to convert the fundamental to the second harmonic. By numerically solving these
rate equations, a group of general curves are obtained to express the relations between the solution
and the three dimensionless parameters. In addition, the optimal pump to laser-mode size ratio
and the optimal $\eta_{\text{SHG}}$ are determined. A comparison with the result obtained under the plane-wave
approximation is also given.

1 The rate equations

When a laser beam with an intensity of $I_F(r)$ (where $r$ is the radial coordinate) passes
through a nonlinear crystal with a length of $l_n$, the generated second-harmonic intensity $I_{\text{SH}}(r)$
in the case of low second-harmonic generation (SHG) conversion efficiency \([I_{\text{SH}}(r)/I_F(r) <
20\%] \) can be written as\cite{13}
\[
I_{\text{SH}}(r) \approx \eta \frac{l_r^2(r)}{r},
\]
where $\eta$, a parameter to mark the ability of the nonlinear crystal to convert the fundamental to the
second harmonic, can be written as
\[
\eta = 2 \left( \frac{\mu_0}{\varepsilon_0} \right)^{1/2} \frac{\omega^2 d_{\text{eff}}^2 l_n^2 \sin^2(l_n\Delta k/2)}{r_1^2 r_2 c^2 (l_n\Delta k/2)^2},
\]
where $\omega$ is the radian frequency of the fundamental, $d_{\text{eff}}$ (in MKS units) is the effective nonlinear
coefficient of the nonlinear crystal, $\mu_0$ and $\varepsilon_0$ are the permeability and permittivity of free space,
respectively, $r_1$ and $r_2$ are the refractive indexes of the nonlinear crystal for the fundamental and
the second harmonic, respectively, $c$ is the light speed in vacuum, and $\Delta k$ is the phase
mismatch.

For laser operating in the TEM$_{00}$ mode, the intracavity photon density $\phi_g(r)$ (in free space)
can be written as
\[
\phi_g(r) = \phi_{g0} \exp \left( - \frac{2r^2}{\omega_{1L}^2} \right),
\]
where $\phi_{g0}$ is the photon density in the laser axis, and $\omega_{1L}$ is the laser spot radius in the gain.